Mergers, remedies and efficiency gains [*]

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Abstract. In this paper, we analyse the combination of structural remedies and efficiency gains that may lead to a pro-competitive merger. We show that two types of efficiencies are necessary. The first corresponds to a less steep marginal cost, the second to a lower intercept of the marginal cost curve. If this second type of efficiency gains is not sufficient to make the post-merger price falling, then divestitures are adopted, and we compute the amount of divested asset proposed by the merged entity. This paper allows us to compare two kinds of divestitures and we show that if the divested capital is distributed among the outsiders, the merger is more easily pro-competitive than if divested capital is sold to a single competitor.

Keywords: Mergers; Efficiency Defence; Divestitures.

JEL classification: L4; K21; D43.

1 Introduction

In the European Union, merger control has evolved in several important ways in recent years, for example in the development of a policy on remedies that has been codified in a "Remedies Notice"[1]. The European Community merger regulation refers to acceptable remedies as modifications to concentrations designed to reduce the merging parties' market power, to avoid the creation of a dominant position and to restore conditions for effective competition [2].

The following proposed "SEB-Moulinex" merger provides a good illustration of the use of remedies. On 2002, the European Commission decided to clear the proposed merger between SEB and Moulinex, two French companies, subject to compliance with certain commitments, notably SEB must grant third parties an exclusive licence to the mark Moulinex for a period of 5 years in 9 member States of the European Economic area and SEB must abstain from using the mark Moulinex for three years following the expiry of these licences. However, Babyliss, a French company, brought a case before the Court of First Instance against the decision of the Commission. Moreover, Philips, a Dutch company, also brought a case before the Court of First Instance requesting the annulment of the merger decision and contested the referral to the French authorities. On April 2003, the Court of First Instance annulled the Commission's decision with respect to five countries where no remedies has been required. The Commission then reopened the first phase of investigation and concluded that the commitments offered by SEB did not overcome the serious doubts as to the compatibility of the transaction with the common market.

Merger remedies can be classified into two categories : structural and behavioural remedies. The first can be prohibition, dissolution or partial divestiture. Prohibition or dissolution involves preventing the merger in its entirety. Partial divestiture might be required to eliminate identified anti-competitive effects. The second category of merger remedies is modification of the behaviour of the merged firm in order to prevent or reduce anti-competitive effects. This can be achieved through a variety of one-time conditions and ongoing requirements. Behavioural remedies require ongoing regulatory oversight and intervention. Structural remedies are often more likely to be effective in the long run and require less ongoing governmental intervention. Competition authorities generally prefer structural remedies over behavioural remedies because the terms of such remedies are clearer

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[1]Commission notice on remedies acceptable under council regulation 4064/89, OJ 2001 C 68/3.

[2]Council regulation (EEC) No 4064/89 and commission regulation (EC) No 447/98.

and certain, are less costly to administer, and are readily enforceable.

When a competition authority faces a potentially anticompetitive merger, it is faced with three choices : either unconditionally accepting the merger according to efficiency gains ("efficiency defence"), blocking it or approving it but subject to remedial actions.

There are two competing welfare standards in antitrust economics: the total welfare standard and the consumer welfare standard. The difference is that the total welfare standard seeks to protect consumers by way of maximizing welfare for society, whereas the consumer welfare standard treats consumers as the end goal of antitrust. The welfare standard used in the application of the merger control legislation in jurisdictions such as the US and the EU are strongly biased in favour of consumers. Therefore, in this paper, we suppose that the antitrust authority accepts a merger only if it satisfies the price standard. The price standard requires that the post-merger price in the relevant market not exceed the pre-merger price or, equivalently, that there is no decrease in consumer surplus :

$$\Delta p \leq 0 \Leftrightarrow \Delta S^c \geq 0$$

where Δp and ΔS^c are the price variation and the consumer surplus variation between pre-merger and post-merger situation.

The term "efficiencies" is not formally defined in the EC merger regulation or in the horizontal merger guidelines. What constitutes an efficiency gain rather than loss depends on the welfare standard that one chooses. Bian and McFetridge (2000) compare alternative interpretations on the efficiency defence and find that when spillovers are low and markets are concentrated, the efficiency gains required to satisfy the price standard are very high in absolute terms and are four or more times higher than is required for a profitable, total-surplus-increasing merger.

Some studies examine and analyse the role of remedies in merger control. Using a spatially differentiated oligopoly with free-entry, Cabral (2002) studies the effects of a merger between two multi-location firms to show that a more efficient merged firm (in the sense of lower marginal cost) implies a lower equilibrium price but also a less likely entry in the industry. The benefit that consumers receive from cost efficiencies is therefore lower than the benefit which they would receive if entry conditions were not exogenous. He also studies the effect of divestiture and shows that by selling stores to potential rivals, merging firms may effectively buy them off, that is, dissuade them from opening new stores which is bad for consumers.

Goppelsroeder, Schinkel and Tuinstra (2006) create an index based on

compensating marginal cost reductions[3] to measure the average reduction in marginal costs required to restore pre-merger equilibrium prices and output after the merger is consummated. The index integrates concentration and efficiency effects, and asks what average percentage of total (variable) cost savings need minimally materialize as a result of the merger to compensate consumers for its anticompetitive effects.

Our paper is most closely related to works by Medvedev (2004) and Vasconcelos (2005). Medvedev (2004) analyses equilibrium of a Cournot market before and after a merger with a focus on profitability and welfare changes. Medvedev works on the basis that the antitrust agency will apply a consumer surplus standard and will approve mergers that decrease prices, while rejecting those that increase prices. He shows that given a pre-merger symmetric cost structure, any divestiture leads to lower prices than would prevail without divestiture. The second related paper is Vasconcelos (2005) who analyses effects of endogenous mergers in a Cournot setting. Contrary to Medvedev he considers only a discrete number of possible divestitures which can be selected and outsiders may be pushed out of the industry if the merger is approved. He reports that the required divestitures of assets induce a more competitive outcome (in terms of lower equilibrium prices) after the merger-plus-divestiture than prevailed before in the status quo industry structure.

Our paper considers a Cournot industry with homogeneous good where each firm competes with the same quadratic function cost. We study the price effect of a merger and we take account of two kinds of efficiencies. We call "efficiency gains of category I" the fact that a merger brings the individual capital of the merging firms under a single larger resulting firm. "Efficiency gains of category II" are merger-specific synergies between merging firms' assets, they decrease the parameter c in the cost function.

Moreover, we allow for divestiture. Firstly, we suppose that merged firms divest assets to only one other firm. After that, we extend the model by another kind of divestiture in which merged firms divest the same amount of assets to each firm in the industry. The first kind of divestiture requires less ongoing government intervention because it implies the merged entity and only one other firm.

We find that merger which does not generate efficiency gains of category II raises prices, so the merger is rejected by antitrust authority. We show that if the merger-efficiencies are large enough, then the anticompetitive effect of the merger can be counteracted. If however the efficiency gains are too small, then divestiture is needed, which can in turn reduce post-merger prices.

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[3]this concept was developed in Werden (1996) and Froeb and Werden (1998)

Moreover we prove that a merged entity not generating enough efficiency must divest a minimal amount of assets but they do not have to divest too much. Finally, we establish that by making the industry more symmetric, post-merger prices with divestiture to each firm are always lower than post-merger prices with divestiture to only one firm.

The article is organized as follows. In section 2 we describe the basic model. Characterizations of equilibria and implications for merger control are provided in section 3. In section 4, we extend the analysis specifying an other procedure of divestiture. Concluding remarks follow in section 5. Proofs of results appear in the appendix.

2   The model

We consider a simple framework consisting of a Cournot homogenous product industry. Demand is linear, with price $p = a - Q$ ($a > 0$) where Q denotes the total quantity produced in the industry. We assume that n firms are active in this market. Each firm has a share of a specific asset that affects marginal costs. More precisely each firm has the same production capacity, denoted by k_i. The total supply of capital in the industry is assumed to be fixed, which is normalized to be one: $\sum_{i=1}^{n} k_i = 1, \forall i = 1...n$.

Following Perry and Porter (1985), cost is assumed to be dependant on the capital owned by the firms :

$$C(q_i, k_i) = cq_i + \frac{q_i^2}{2k_i}, \forall i \in [1, n]$$

where $0 < c < 1$, $a > c$, $0 < k_i < 1$ and q_i denotes the quantity chosen by firm i. Fixed costs are zero[4]. We assume an upward-sloping linear marginal cost function, where the slope is a decreasing function of the firms' share of the industry's stock of capital :

$$\frac{\partial C(q_i, k_i)}{\partial q_i} = c + \frac{q_i}{k_i} > 0 \;\; ; \;\; \frac{\partial^2 C(q_i, k_i)}{\partial q_i^2} > 0 \;\; ; \;\; \frac{\partial^2 C(q_i, k_i)}{\partial q_i \partial k_i} < 0$$

We focus on the effects of divested assets on the price after a merger between two firms, namely firms 1 and 2. Firstly, we suppose that merged firms divest assets to only one firm in the industry. The volume of divested assets is denoted by k ($0 \leq k < k_i$). In the last section, we extend the

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[4]efficiency gains on fixed cost have no impact on equilibrium price, only efficiency gains affecting variable costs can impact.

model authorizing to divest the same proportion of assets to each firm in the industry.

Moreover, mergers efficiencies can be twofold. The first are mechanical efficiency gains: when two firms merge, they have double production capacity ($2k_i$) and the merged entity faces a slowly rising marginal cost curve. So any merger gives rise to efficiency gains since it brings the individual capital of the merging firms under a single larger resulting firm. Mechanical efficiencies are referred by "efficiency of category I". Furthermore, there could be efficiency gains due to merger-specific synergies between merging firms' assets. We introduce this by decreasing the parameter c in merged firm cost function by a value e ($0 < e \leq 1$). This is what we call an "efficiency of category II". So cost function of the merged entity is:

$$C(q_m, k_m) = ecq_m + \frac{q_m^2}{2k_m}, 0 < e \leq 1$$

where q_m and $k_m(k_m = 2.k_i)$ design respectively the quantity and the production capacity of the merged entity.

The following graphic represents how the marginal cost curve moves with each type of efficiency gain. The "efficiency of category I" is represented by changing the slope of the marginal curve. The "efficiency of category II" is represented by moving down the intercept of the marginal cost curve.
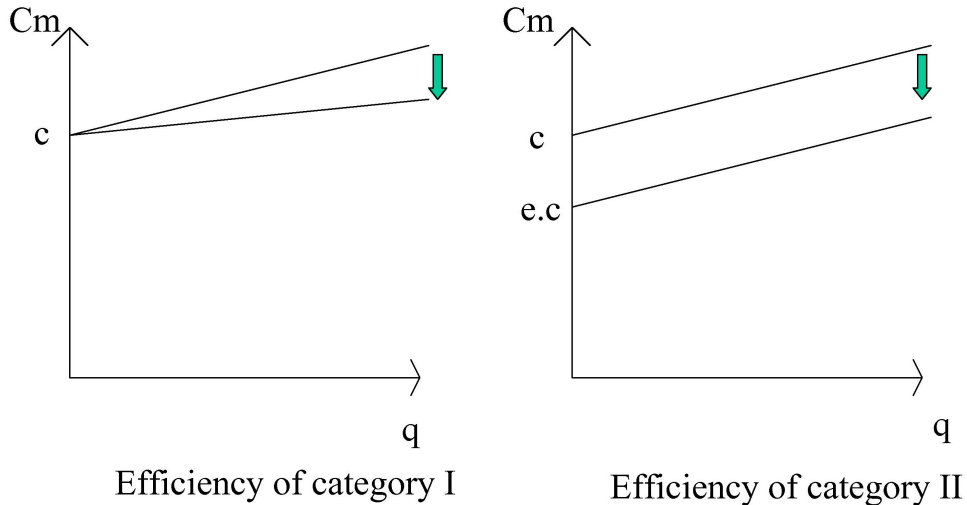

Figure 1: Moving of marginal cost depending on the category of efficiency

There are four possibilities to represent the merger between firms 1 and 2:

- Situation 1: "simple" merger with efficiency of category I, but without efficiency of category II and without divestiture.

- Situation 2: merger without divestiture, but with efficiency gains of categories I and II.

- Situation 3: merger with only efficiency gains of category I and divestiture.

- Situation 4: merger with efficiency gains of categories I and II and divestiture.

The following table summarizes the four situations.

	Efficiency of category I	Efficiencies of categories I and II
Without divestiture	Situation I	Situation II
With divestiture	Situation III	Situation IV

Table 1: four possibilities to represent the merger

In our paper, efficiency gain and divestiture are required to satisfy price standard. The price standard requires that the post-merger price in the relevant market not exceed the pre-merger price or, equivalently, that there is no decrease in consumer surplus. First, we study a merger's effect on price in each situation. Second, we compute the amount of divestiture k (situations III and IV) and the value of efficiency of category II (e) required to satisfy the price standard.

More precisely, when they merge, firms know the value of efficiency gains "e" and depending on this value, they have to divest assets or not.

3 Divestiture to only one firm

Once introduced the framework, we can address mergers related issues. In this section, we suppose that the merged entity can divest assets to only one active firm in the industry.

3.1 Equilibrium prices

Equilibrium prices in each different situation in the industry are given in the Table 1[5]. Firms are *ex ante* symmetric so $k_i = 1/n$.

Before merger, equilibrium price (p^c), is given by :

$$p^c = \frac{a + n(a + c)}{1 + 2n}$$

Proof. See appendix A.   □

After merger, equilibrium prices are given by the following table:

Table 1 : Equilibrium prices

Situation	Equilibrium prices
Situation 1	$\frac{(2+n)(a+n(a+c))-2c}{n(5+2n)}$
Situation 2	$\frac{a(n+1)(n+2)+c(n^2+2e(n+1)-4)}{n(5+2n)}$
Situation 3	$\frac{a(1+n)(-2+n(k-1))(1+n+kn)+c(2+n(k-3kn-n(3+n)+k^2n(3n-1)))}{Y}$
Situation 4	$\frac{X}{Y}$

with
$X = a(1+n)(-2+(-1+k)n)(1+n(k+1)) + c(e(1+n)(-2+kn)(1+n(k+1)) + (-2+(-1+k)n)(-2+(1+2k)(-1+n)n))$
and
$Y = n(-5 + n(-7 - 2n + 4k(-1 + kn)))$

Proof. See appendix A.   □

When we compare the price before merger with after merger, we obtain the following proposition.

proposition 1. *Without divestiture, a merger which does not generate efficiency gains of category II results in a price increase.*

Proof. See Appendix B.   □

This result reinforces the results obtained by McAfee and Williams (1992) who showed that, using a quadratic cost function (as Perry and Porter (1985) and a linear demand, a merger between two firms causes total output and consumer surplus to fall except if there is a non-merging firm whose market

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[5]Proof in appendix A

share exceeds the sum of the pre-merger shares of the merging firms. Farrell and Shapiro (1990) showed that in a Cournot industry with homogeneous goods, any merger not creating synergies[6] raises price. More precisely, a merger can raise output (consequently can decrease price) only if there exist economies of scale between merging firms or if the merging firms learn form each other (for instance sharing techniques...).

Much more interesting is to compare the pre-merger situation with the post-merger situation with divestiture.

proposition 2. *Whatever the amount of divestiture to one firm is (with $0 \leq k < k_i$), post-merger price is strictly greater than the competitive price that prevailed before merger if it does not generate efficiency gains of category II.*

Proof. See Appendix C. $\square$

Consequently, divestiture cannot be sufficient to make global output rise instead of fall if the merger does not generate efficiency gains of category II.

In the next subsection, we illustrate how large these efficiency gains must be for a merger to lower price.

3.2 Price standard

The price standard requires that the post-merger price in the relevant market not exceed the pre-merger price or, equivalently, that there is no decrease in consumer surplus. The following results are obtained by comparing pre and post-merger prices.

Let p^c, p_{eff}^{merger} and $p_{eff+div}^{merger}$ denote the prices before merger (competitive price), after merger with efficiency gains of categories I and II and after merger with efficiency gains of categories I and II and divestiture.

We compute the minimum value of efficiency gains of category II (e) for a merger to increase output and reduce price :

lemma 1.

- $p^c \leq p_{eff}^{merger} \quad \Leftrightarrow \quad e \geq e_1$

- $p^c \leq p_{eff+div}^{merger} \quad \Leftrightarrow \quad e \geq e_2$

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[6]No synergies refers to a situation where production possibilities of the merged entity are not different from those of the insiders jointly before the merger.

with

$$\begin{cases} e_1 = \frac{-a+2c(n+1)}{c(1+2n)} \\ e_2 = \frac{a(2+n(2+k(2n-1)(kn-1)))+c(-4+n(-4(2+n)+k(n+2(n^2+kn-14))))}{c(1+2n)(kn-2)(1+n(k+1))} \end{cases}$$

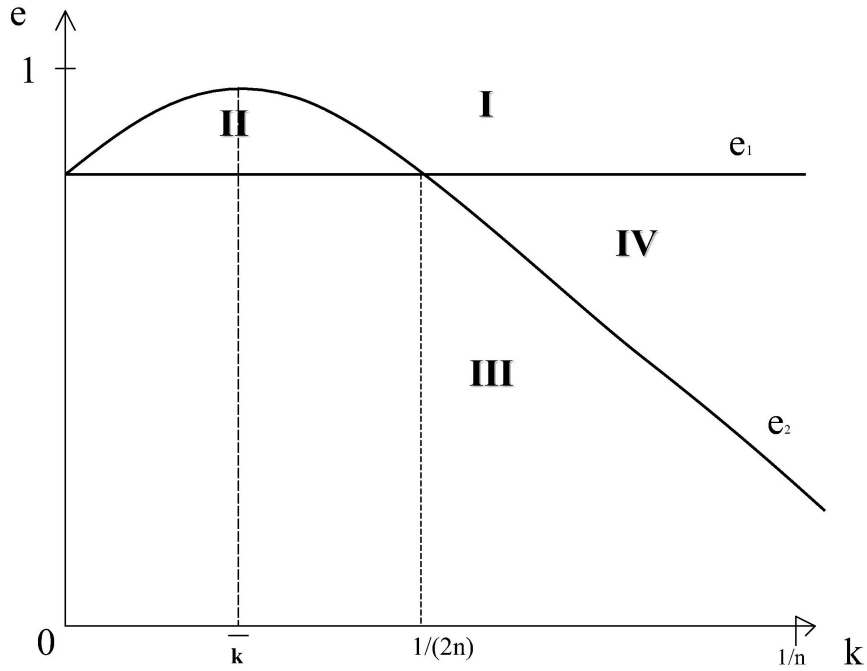These functions are plotted on figure 2[7].

Figure 2: Graphical representation of e_1 and e_2

Description of the regions:

- Region **I** : $p^c < p_{eff}^{merger}$ and $p^c < p_{eff+div}^{merger}$

- Region **II** : $p_{eff+div}^{merger} < p^c < p_{eff}^{merger}$

- Region **III** : $p^c > p_{eff}^{merger}$ and $p^c > p_{eff+div}^{merger}$

- Region **IV** : $p_{eff}^{merger} < p^c < p_{eff+div}^{merger}$

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[7]Proof are reported in Appendix D

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In the region I, structural remedies are not effective to maintain or decrease price after merger. In the region II, remedies are necessary. In the region III, using structural remedies is not necessary because efficiency gains of category II are sufficient to maintain or decrease post-merger price. An interesting case is the region IV in which merger decreases price without any divestiture but can increase it with divestiture. This is because if $k > \frac{1}{2n}$ the merged firm is not the biggest firm in the industry, the firm which bought divested assets becomes bigger. More precisely, if we are in region IV then : $k > \frac{1}{2n}$ so capacity production of the merged firm is : $2k_i - k < \frac{2}{n} - \frac{1}{2n}$, capacity production of the firm which bought assets is : $k_i + k > \frac{1}{n} + \frac{1}{2n}$ and capacity of each outsider firm is : $k_i = \frac{1}{n}$. The following table summarizes the situation in the region IV :

	merged entity	firm which bought divested assets	outsiders
Before merger	$2k_i$	k_i	k_i
After merger	$2k_i - k$	$k_i + k$	k_i

Table 2: Production capacities before and after merger

We can demonstrate that if: $k > \frac{1}{2n}$ then $2k_i - k < k_i + k$, and merged firm is not the biggest firm in the industry. Moreover, efficiency applies to this firm which implies that price increases because the total quantity in the market decreases [8].

e_2 is a concave function which is strictly increasing for values of k strictly less than $\bar{k}$ and strictly decreasing for $k > \bar{k}$ (where $\bar{k}$ is obtained by the first order condition $\frac{\partial e_2}{\partial k} = 0$)[9]. This concavity implies that if the amount of divestiture increases, but without exceeding $\bar{k}$, then e has to increase in order to maintain the following equality: $p^c = p^{merger}_{eff+div}$. In other words, the amount of efficiency gains decreases (e increases) in order to maintain constant price before and after merger. But, for values of k strictly greater than $\bar{k}$, then if the amount of divestiture increases, e has to decrease in order to maintain the previous equality.

We now study situations in which divestiture is necessary for the merger to be accepted (region II).

Let

$$\epsilon_2 = \underbrace{Max}_{k} e_2(k), \forall k \in [0, \frac{1}{n}]$$

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[8]Proof : see proposition 4

[9]See Appendix D

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and
$$e \in (e_1, \epsilon_2]$$

For $\underline{e} \in (e_1, \epsilon_2]$, we compute the values of k such that price decreases after merger :
$$\Delta p \leq 0$$

with
$$\Delta p = p^{merger}_{eff+div} - p^c$$

We can prove that $\Delta p \leq 0$ for $k \in [k_1, k_2]$ [10].

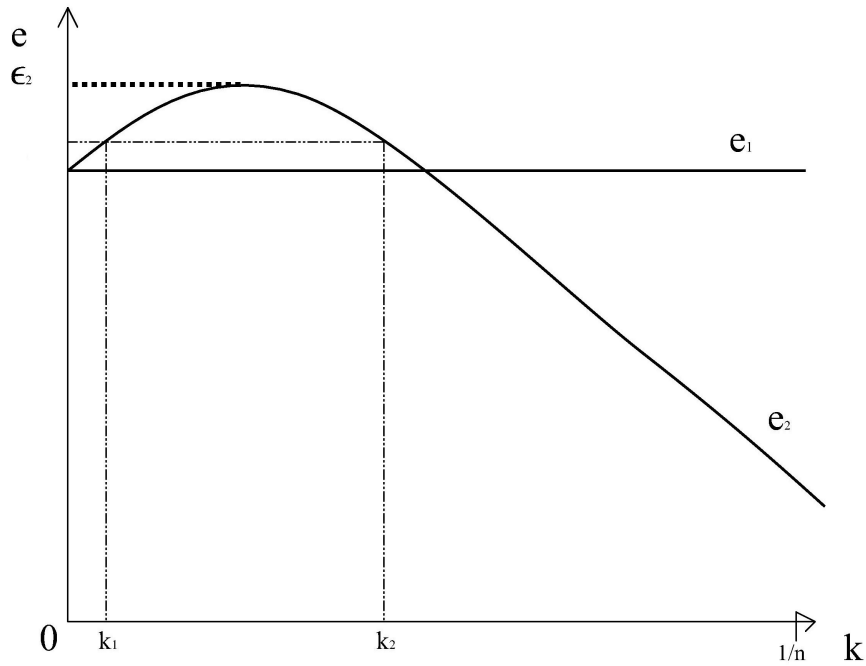The previous situation is represented in the following graphic :

Figure 3: Graphical representation of k_1, k_2 and ϵ_2

The merger between firms 1 and 2 has two major implications. Firstly, market power of the merged entity is increased. Secondly, efficiency gains are generated by the merger. We must determine which is the predominant effect.

The following proposition determine the necessary conditions for divestiture:

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[10] The values of k_1 and k_2 are reported in appendix E

proposition 3. *Divestiture conditions*

- *If $e \in (e_1, \epsilon_2]$ then divestiture is necessary to decrease post-merger price and the amount of divestiture is given by : $k \in [k_1, k_2]$.*

- *If $e \leq e_1$ then efficiency gains are sufficient not to increase prices. Divestiture is not necessary.*

- *If $\epsilon_2 < e < 1$ then merger increases price.*

If the merger-efficiencies are large enough ($e \leq e_1$), the anticompetitive effect of the merger can be counteracted and the merger must be accepted. Thus, the post-merger price depends on which effect dominates, the market power effect or the efficiency effect. If the merger generates too little efficiency ($e > e_1$) then two cases can arise. The first is when divestiture can decrease post-merger price and consequently the merger is accepted depending on the proposed amount of divestiture. The second is when whatever the amount of divestiture is, merger increases price and therefore merger is strictly rejected.

corollary 1. *For a given amount of efficiency ($e \in (e_1, \epsilon_2]$), firms have to divest a minimal amount ($k \geq k_1$) but they do not have to divest too much ($k \leq k_2$) to decrease post-merger price.*

Firms have to divest assets (depending on the value of efficiency gains) to decrease price but not too much. To understand this, we have to make comparative statics on equilibrium quantity.

We denote by q_m^* the equilibrium quantity of the merged entity, by q_D^* the equilibrium quantity of the firm which has bought the assets and by q_i^* the individual equilibrium quantity of the other firms.

We obtain the following proposition:

proposition 4.

- $\frac{\partial q_m^*}{\partial e} < 0$, $\frac{\partial q_i^*}{\partial e} > 0$, $\frac{\partial q_D^*}{\partial e} > 0$, $\frac{\partial Q^*}{\partial e} < 0$

- $\frac{\partial q_m^*}{\partial k} < 0$, $\frac{\partial q_i^*}{\partial k} > 0 \Leftrightarrow k > \hat{k}$, $\frac{\partial q_D^*}{\partial k} > 0$, $\frac{\partial Q^*}{\partial k} > 0 \Leftrightarrow k < \hat{k}$

  with
  $$\hat{k} = -\frac{n(1+n)(a(-3-2n) + c(6 + e(-3+2n))) + \sqrt{C}}{4c(e-1)n^3}$$
  $C = (n^2(1+n)((1+n)(c(-6 + e(3-2n)) + a(3+2n))^2 - 4c(e-1)n(a(3+2n) + c(-6 - n(5+2n) + e(3 + n(3+2n))))))$

Proof. See appendix F. □

When efficiency gains become smaller (e is increasing and cost function increases), total quantity decreases and this is anticompetitive. Moreover, if we increase the amount of divestiture without divesting more than $\hat{k}$, then total quantity increases and this effect can counteract the anticompetitive effect (k and e are strategic substitutes). But if we divest too much ($k > \hat{k}$), total quantity decreases and reinforces the anticompetitive effect of the decreasing in efficiency gains (k and e become strategic complements).

The minimal and the maximal divest amount required to decrease price depend on the value of efficiency gains. Proposition 5 follows:

proposition 5. *The minimal divest amount required to decrease price increases with e whereas the maximal amount decreases with e.*

The proof is caused by the concavity of the function e_2.

3.3 Proposed amount of divestiture by merged parties

For $e \in (e_1, \epsilon_2]$, the merged entity has to propose an amount of divestiture between k_1 and k_2 previously defined.

The equilibrium profit of the merged entity is given by :

$$\pi^*_{eff+div} = \frac{(-2 + kn)(-4 + (-1 + 2k)n)W}{2n^2(-5 + n(-7 - 2n + 4k(-1 + kn)))^2}$$

with

$$W = (a(1 + n)(1 + n + kn) + c(-2 + e + e(k - 1)n + (1 + 2k)(n - 1)n - e(2 + 3k)n^2))^2$$

Numerical simulations show that this profit function is decreasing for every $k \in ([0, 1/n]$.

Consequently, merged entity which generates efficiency gains between e_1 and ϵ_2 chooses to propose the minimal amount of divestiture required not to increase price.

proposition 6. *If merger efficiencies are not large enough to decrease price, firms choose to divest the minimal amount required to satisfy the price standard.*

4 Divestiture to all firms

In this section, we now consider the case in which the merged entity divests the same amount of assets to each firm in the industry.

Proposition 2 still applies for this kind of divestiture.

In this case, the post-merger price with divestiture and efficiency of categories I and II is given by :

$$p = \frac{a(-2 + (-1 + k)n)(-2 + n(-1 + k + n)) + c*Z}{n(10 + n(-1 - 7k + (-2 + k(2 + k))n))}$$

with $Z = (n - 2)(-2 + (k - 1)n)(-2 + n + kn) + e(kn - 2)(-2 + n(k + n - 1))$.

Proof. See appendix G. □

Comparing post-merger price with efficiency according to the two kinds of divestiture, we obtain the following proposition :

proposition 7. *By making the industry "more symmetric", post-merger price with divestiture to each firm is always lower than post-merger price with divestiture to only one firm.*

Proof. See appendix H. □

Divestiture to each firm makes the industry more symmetric. This implies the decreasing of unilateral effects, but generally, symmetry increases the possibility for firms to make collusive agreements. This is a dilemma faced by the antitrust authority : to increase symmetry (in order to decrease price) or to decrease it (in order to decrease the risk of coordinated effects).

We compute the minimum value of efficiency for a merger to increase output and reduce price and we obtain the following lemma :

lemma 2. $p^c \leq p_{eff+div}^{merger}$ if $e \geq e_3$

with :

$$e_3 = \frac{(-4a + 8c - 2an + 12cn + 4akn - 8ckn + 2an^2 + 3akn^2 - 12ckn^2 - ak^2n^2)}{(c(1 + 2n)(4 + 2n - 4kn - 2n^2 - kn^2 + k^2n^2 + kn^3))} +$$
$$\frac{2ck^2n^2 - 4cn^3 - 2akn^3 + ckn^3 - ak^2n^3 + 3ck^2n^3 + 2ckn^4 + ak^2n^4 - ck^2n^4}{(c(1 + 2n)(4 + 2n - 4kn - 2n^2 - kn^2 + k^2n^2 + kn^3))}$$

If we plot the function e_3 in the figure 1, we obtain the following graphic :
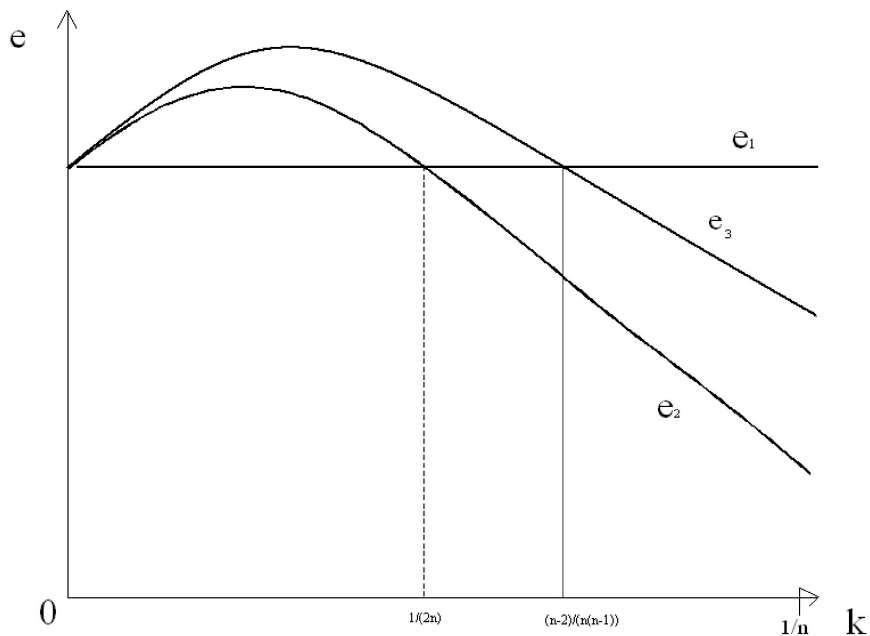
Figure 4: Graphical representation of e_1, e_2 and e_3

Proof. See appendix I. □

The minimal divest amount required to decrease price is lower in the case of divestiture to each firm than to only one firm whereas the maximal amount is higher.

5   Concluding remarks

The results of the present paper derive some enforcement policy implications, which we summarize in this section.

First, we prove that the antitrust authority has to approve a merger between two firms if efficiency gains of category II are high enough because it satisfies the price standard. A merger which does not generate the two categories of efficiency gains must be rejected because the post-merger price will always be greater than the pre-merger price. Moreover, if the merger generates not enough efficiency, then firms must divest a minimal amount of assets to decrease post-merger price but they do not have too divest too

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much. Finally, we prove that to divest all assets to only one firm is less beneficial than to proportionally divest assets to each firm in the industry.

Although distributing the divested capital among all the outsides is preferable, this restructuring might be unfeasible in practice. The Commission Notice on remedies stresses that the assets divested have to create a viable competitive entity, and it is sceptical about mix-and-match divestitures that fragment too much the assets[11]. Then selling all the divested assets to a single buyer might be preferable on this ground.

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[11]Motta, Polo and Vasconcelos (2005)

6 Appendix

Appendix A : Equilibrium prices

In the "no merger situation", the maximization program is:

$$Max_{q_i}(a - Q)q_i - cq_i - \frac{q_i^2}{2k_i}, \forall i = 1..n$$

We obtain a symmetric equilibrium :

$$q_i = q^* = \frac{a - c}{1 + 2n}, \forall i = 1..n$$

and then $p^* = a - nq_i^*$.

Let design by q_M the quantity of the merged entity and by q_D the quantity of the firm which bought divested assets.

In the "simple merger situation", the maximization program is :

$$\begin{cases} Max_{q_i}(a - Q)q_i - c*q_i - \frac{q_i^2}{2k_i}, \forall i \in out \\ Max_{q_M}(a - Q)q_M - c*q_M - \frac{q_M^2}{4k_i} \end{cases}$$

First Order Conditions:

$$\begin{cases} \frac{\partial \pi_i}{\partial q_i} = 0, \forall i \in out \\ \frac{\partial \pi_M}{\partial q_M} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} q_i = \frac{a - c - q_M - (n-3)q_i}{2 + k_i^{-1}}, \forall i \in out \\ q_M = \frac{a - c - (n-2)q_i}{2 + (2k_i)^{-1}} \end{cases}$$

Intersection of the best-response functions yield:

$$\begin{cases} q_i^* = \frac{(a-c)(2+n)}{n(5+2n)}, \forall i \in out \\ q_M^* = \frac{2(a-c)(n+1)}{n(5+2n)} \end{cases}$$

$q_i^* > 0$ and $q_M^* > 0$ iff $a - c > 0$

And: $p^* = a - Q^*$.

Second Order Conditions:

$$\begin{cases} \frac{\partial^2 \pi_i}{\partial q_i^2} < 0, \forall i \in out \\ \frac{\partial^2 \pi_M}{\partial q_M^2} < 0 \end{cases}$$

$\Rightarrow$
$$\begin{cases} -2 - \frac{1}{k_i} < 0 \\ -2 - \frac{1}{2k_i} < 0 \end{cases}$$

In the "merger+efficiency situation", the maximization program is :

$$\begin{cases} Max_{q_i}(a - Q)q_i - cq_i - \frac{q_i^2}{2k_i}, \forall i \in out \\ Max_{q_M}(a - Q)q_M - \mathbf{e}cq_M - \frac{q_M^2}{2(2k_i)} \end{cases}$$

First Order Conditions:

$$\begin{cases} \frac{\partial \pi_i}{\partial q_i} = 0, \forall i \in out \\ \frac{\partial \pi_M}{\partial q_M} = 0 \end{cases}$$
$\Rightarrow$
$$\begin{cases} q_i = \frac{a - c - q_M - (n-3)q_i}{2 + k_i^{-1}}, \forall i \in out \\ q_M = \frac{a - ec - (n-2)q_i}{2 + (2k_i)^{-1}} \end{cases}$$

Intersection of best-response functions yield:

$$\begin{cases} q_i^* = \frac{-a(2+n) + c(4 - 2e + n)}{n(5 + 2n)}, \forall i \in out \\ q_M^* = \frac{2(a(n+1) + c(-2 + e + n - 2en))}{n(5 + 2n)} \end{cases}$$
And: $p^* = a - Q^*$.

Second Order Conditions:

$$\begin{cases} \frac{\partial^2 \pi_i}{\partial q_i^2} < 0, \forall i \in out \\ \frac{\partial^2 \pi_M}{\partial q_M^2} < 0 \end{cases}$$
$\Rightarrow$
$$\begin{cases} -2 - \frac{1}{k_i} < 0 \\ -2 - \frac{1}{2k_i} < 0 \end{cases}$$

In the "merger+divestiture situation", the maximization program is :

$$\begin{cases} Max_{q_i}(a-Q)q_i - cq_i - \frac{q_i^2}{2k_i}, \forall i \in out \\ Max_{q_M}(a-Q)q_M - cq_M - \frac{q_M^2}{2(2k_i\text{-}\mathbf{k})} \\ Max_{q_D}(a-Q)q_D - cq_D - \frac{q_D^2}{2(k_i+\mathbf{k})} \end{cases}$$

First Order Conditions:

$$\begin{cases} \frac{\partial \pi_i}{\partial q_i} = 0, \forall i \in out \\ \frac{\partial \pi_M}{\partial q_M} = 0 \\ \frac{\partial \pi_D}{\partial q_D} = 0 \end{cases}$$
$$\Rightarrow$$
$$\begin{cases} q_i = \frac{a-c-q_M-q_D-(n-4)q_i}{2+k_i^{-1}}, \forall i \in out \\ q_M = \frac{a-c-(n-3)q_i-q_D}{2+(2k_i-k)^{-1}} \\ q_D = \frac{a-c-(n-3)q_i-q_M}{2+(k_i+k)^{-1}} \end{cases}$$

Intersection of the best-response functions yield:

$$\begin{cases} q_i^* = \frac{(a-c)(-2+n(k-1))(1+n+kn)}{n(-5+n(-7-2n+4k(kn-1)))}, \forall i \in out \\ q_M^* = \frac{(a-c)(n+1)(kn-2)(1+n+kn)}{n(-5+n(-7-2n+4k(kn-1)))} \\ q_D^* = \frac{(a-c)(n+1)(-2+(k-1)n)(1+kn)}{n(-5+n(-7-2n+4k(kn-1)))} \end{cases}$$
And: $p^* = a - Q^*$.
Second Order Conditions:

$$\begin{cases} \frac{\partial^2 \pi_i}{\partial q_i^2} < 0, \forall i \in out \\ \frac{\partial^2 \pi_M}{\partial q_M^2} < 0 \\ \frac{\partial^2 \pi_D}{\partial q_D^2} < 0 \end{cases}$$
$$\Rightarrow$$
$$\begin{cases} -2 - \frac{1}{k_i} < 0 \\ -2 - \frac{1}{2k_i-k} < 0 \\ -2 - \frac{1}{k_i+k} < 0 \end{cases}$$

In the "merger+divestiture+efficiency situation", the maximization program is :

$$\begin{cases} Max_{q_i}(a-Q)q_i - cq_i - \frac{q_i^2}{2k_i}, \forall i \in out \\ Max_{q_M}(a-Q)q_M - \mathbf{e}cq_M - \frac{q_M^2}{2(2k_i-\mathbf{k})} \\ Max_{q_D}(a-Q)q_D - cq_D - \frac{q_D^2}{2(k_i+\mathbf{k})} \end{cases}$$

First Order Conditions:

$$\begin{cases} \frac{\partial \pi_i}{\partial q_i} = 0, \forall i \in out \\ \frac{\partial \pi_M}{\partial q_M} = 0 \\ \frac{\partial \pi_D}{\partial q_D} = 0 \end{cases}$$

Intersection of the best-response functions yield:

$$\Rightarrow$$
$$\begin{cases} q_i = \frac{a-c-q_M-q_D-(n-4)q_i}{2+k_i^{-1}}, \forall i \in out \\ q_M = \frac{a-ec-(n-3)q_i-q_D}{2+(2k_i-k)^{-1}} \\ q_D = \frac{a-c-(n-3)q_i-q_M}{2+(k_i+k)^{-1}} \end{cases}$$
$$\begin{cases} q_i^* = \frac{(1+n+kn)(-2(a+c(e-2))+(c+a(k-1)+c(e-2)k)n)}{n(-5+n(-7-2n+4k(kn-1)))}, \forall i \in out \\ q_M^* = \frac{(kn-2)(c(e-2)+a(n-1)(1+n+kn)+cn((1+2k)(n-1)+e(-1+k-2n-3kn)))}{n(-5+n(-7-2n+4k(kn-1)))} \\ q_D^* = \frac{((1+n)(1+kn)(-2(a+c(e-2))+(c+a(k-1)+c(e-2)k)n)}{n(-5+n(-7-2n+4k(kn-1)))} \end{cases}$$
And: $p^* = a - Q^*$.

Second Order Conditions:

$$\begin{cases} \frac{\partial^2 \pi_i}{\partial q_i^2} < 0, \forall i \in out \\ \frac{\partial^2 \pi_M}{\partial q_M^2} < 0 \\ \frac{\partial^2 \pi_D}{\partial q_D^2} < 0 \end{cases}$$
$$\Rightarrow$$
$$\begin{cases} -2 - \frac{1}{k_i} < 0 \\ -2 - \frac{1}{2k_i-k} < 0 \\ -2 - \frac{1}{k_i+k} < 0 \end{cases}$$

Appendix B : Proof of proposition 1

We compute the value of the difference between pre-merger price and post-merger price without divestiture and without efficiency gains of category II and we obtain: $\frac{-2(a-c)(n+1)}{n(1+2n)(5+2n)}$. This expression is strictly negative iff $a > c$.

Appendix C : Proof of proposition 2

We compute the value of the difference between pre-merger price and post-merger price with divestiture but without efficiency gains of category II and we obtain: $\frac{(a-c)(n+1)(n(2+k(2n-1)(kn-1))+2)}{n(1+2n)(-5+n(-7-2n+4k(kn-1)))}$.

$0 \le k < \frac{1}{n} \Leftrightarrow -1 \le -1 + kn < 0 \Leftrightarrow n(1+2n)(-5+n(-7-2n+4k(kn-1))) < 0$.

$0 \le k < \frac{1}{n} \Leftrightarrow 0 \le k(2n-1) < 2 - \frac{1}{n} \Rightarrow -(2-\frac{1}{n}) \le k(2n-1)(kn-1)$
$\Leftrightarrow \frac{1}{n} \le k(2n-1)(kn-1) + 2$.

So the difference between pre-merger price and post-merger price with divestiture but without efficiency gains of category II is strictly negative iff $a > c$.

Appendix D : Graphical representation of e_1 and e_2

e_1 is not dependant of k so it is just a horizontal line.

$$e_2 = \frac{a(2+n(2+k(2n-1)(kn-1))) + c(-4+n(-4(2+n)+k(n+2(n^2+kn-14))))}{c(1+2n)(kn-2)(1+n(k+1))}$$

The first order condition is: $\frac{\partial e_2}{\partial k} = 0$ for $k = \bar{k}$.

With
$$\bar{k} = \frac{1}{2n + \frac{\sqrt{\frac{3}{2}} n \sqrt{3+5n+2n^2}}{n+1}}$$

Moreover, the second derivative is:
$\frac{\partial^2 e_2}{\partial k^2} = \frac{-2(a-c)n^3 J}{(c+2cn)(kn-2)^3(1+n+kn)^3}$.

With $J = (-10 + n(-16 - 6n + k(6 + n(6 - 12k(n+1) + k^2 n(2n-1)))))$

$a > c$ and $0 \le k < \frac{1}{n}$, consequently the sign of $\frac{\partial^2 e_2}{\partial k^2}$ is the same than the sign of J.

$J = 2k^3 n^4 - k^3 n^3 - 12k^2 n^2 - 12k^2 n^3 + 6n^2(k-1) + 6kn - 16n - 10$.

$0 \le k < \frac{1}{n} \Rightarrow 0 \le 2k^3 n^4 < 2n$ and $0 \le 6kn < 6$.

$\Rightarrow 0 \le 2k^3 n^4 + 6kn < 2n + 6 \Leftrightarrow -16n \le 2k^3 n^4 + 6kn < 2n + 6 - 16n < -14n + 6$.

So J is strictly negative and consequently $\frac{\partial^2 e_2}{\partial k^2} < 0$.

The maximum of e_2 is achieved for $k = \bar{k}$.

So for $k \le \bar{k}$ then $\frac{\partial e_2}{\partial k} \ge 0$ and for $k \ge \bar{k}$ then $\frac{\partial e_2}{\partial k} \le 0$.

Moreover : $e_2 = e_1$ for $k = 0$ and for $k = k^*$ with $k^* = \frac{1}{2n}$.

Appendix E : Values of k_1 and k_2

$\Delta p = 0 \Rightarrow$:

$$k_1 = \frac{an(2n-1) + cn(2 - e - (e+1)n + 2(e-1)n^2) - \sqrt{n^2 * Z}}{2n^2(a(2n-1) - c(-2+e+2ne))}$$

$$k_2 = \frac{an(2n-1) + cn(2 - e - (e+1)n + 2(e-1)n^2) + \sqrt{n^2 * Z}}{2n^2(a(2n-1) - c(-2+e+2ne))}$$

with

$$Z = -8(1+n)(a + c(-2 + e + 2(e-1)n))(a(-1+2n) + c(e-2+2en)) +$$
$$(a - 2an + c(e - 2 + n + en - 2(e-1)n^2))^2$$

Appendix F : Proof of proposition 4

Recall that q_m^* is the equilibrium quantity of the merged entity, q_D^* is the equilibrium quantity of the firm which has bought assets and q_i^* is the individual equilibrium quantity of the other firms.

Equilibrium quantities are given by (solving the maximization program in the "merger+divestiture+efficiency" situation):

$$\begin{cases} q_i^* = \frac{(1+n+kn)(-2-2c(e-2)+(-1+c+k+c(e-2)k)n)}{n(-5+n(-7-2n+4k(-1+kn)))} \\ q_M^* = -\frac{(-2+kn)(-(1+n)(1+n+kn)+c(2-(1+2k)(n-1)n+e(-1+n(1-k+2n+3kn))))}{n(-5+n(-7-2n+4k(-1+kn)))} \\ q_D^* = \frac{(1+n)(1+kn)(-2-2c(e-2)+(-1+c+k+c(e-2)k)n)}{n(-5+n(-7-2n+4k(-1+kn)))} \\ Q^* = q_D^* + q_M^* + (n-3)q_i^* \end{cases}$$

The first part of the proposition is a comparative static on equilibrium quantities in function of e:

$$\begin{cases} \partial_e(q_M^*) = -\frac{c(-2+kn)(-1+n(1-k+2n+3kn))}{n(-5+n(-7-2n+4k(-1+kn)))} \\ \partial_e(q_D^*) = \frac{(1+n)(1+kn)(-2c+ckn)}{n(-5+n(-7-2n+4k(-1+kn)))} \\ \partial_e(q_i^*) = \frac{(1+n+kn)(-2c+ckn)}{n(-5+n(-7-2n+4k(-1+kn)))} \\ \partial_e(Q^*) = -\frac{c(1+n)(-2+kn)(1+n+kn)}{n(-5+n(-7-2n+4k(-1+kn)))} \end{cases}$$

All of these expressions have the same denominator: $n(-5 + n(-7 - 2n + 4k(-1 + kn))$ which is negative. Moreover $0 \le k < 1/n$ so we can conclude that: $\partial_e(q_i^*) > 0$, $\partial_e(q_D^*) > 0$, $\partial_e(q_M^*) < 0$ and $\partial_e(Q^*) < 0$.

The second part of the proposition is a comparative static on equilibrium quantities in function of k.

We have to compute the values of: $\partial_k(q_i^*)$, $\partial_k(q_D^*)$, $\partial_k(q_M^*)$ and $\partial_k(Q^*)$. Expressions are more complicated so we can not solve this analytically. We make 3-D graphs for each expression depending of k and e and in the graphics obtained, we suppress each negative value of the expressions and we obtain the following results:

$$\partial_k(q_D^*) > 0, \partial_k(q_M^*) < 0 \tag{1}$$

The sign of $\partial_k(q_i^*)$ and $\partial_k(Q^*)$ is not always the same.

Therefore, we solve:

$$\partial_k(q_i^*) = 0$$

and

$$\partial_k(Q^*) = 0$$

and we obtain the following results:

$\partial_k(q_i^*) > 0 \Leftrightarrow k > \hat{k}$ and $\partial_k(Q^*) > 0 \Leftrightarrow k < \hat{k}$ with:

$$\hat{k} = -\frac{n(1+n)(a(-3-2n) + c(6 + e(-3 + 2n))) + \sqrt{C}}{4c(e-1)n^3}$$

with $C = (n^2(1+n)((1+n)(c(-6+e(3-2n)) + a(3+2n))^2 - 4c(e-1)n(a(3+2n) + c(-6 - n(5+2n) + e(3 + n(3+2n)))))))$

Appendix G : Proof of the equilibrium price

The maximization program is :
$$\begin{cases} Max_{q_i}(a - Q)q_i - cq_i - \frac{q_i^2}{2(k_i + \frac{k}{n-2})}, \forall i \in out \\ Max_{q_M}(a - Q)q_M - \mathbf{e}cq_M - \frac{q_M^2}{2(2k_i \mathbf{-k})} \end{cases}$$
First Order Conditions:
$$\begin{cases} \frac{\partial \pi_i}{\partial q_i} = 0, \forall i \in out \\ \frac{\partial \pi_M}{\partial q_M} = 0 \end{cases}$$
$$\Rightarrow$$
$$\begin{cases} q_i = \frac{a - c - q_M}{n - 1 + \frac{1}{k_i + \frac{k}{n-2}}}, \forall i \in out \\ q_M = \frac{a - ec - (n-2)q_i}{2 + (2k_i - k)^{-1}} \end{cases}$$
$$\begin{cases} q_i^* = \frac{(-2 + n + kn)(-2(a + c(e-2)) + (c + a(k-1) + c(e-2)k)n)}{n(10 + n(-1 - 7k + (-2 + k(2+k))n))}, \forall i \in out \\ q_M^* = -\frac{(-2 + kn)(a(2 - n(-1 + k + n)) + c(-(n-2)(-2 + n + kn) + e(2 - (5+k)n + (2+k)n^2)))}{n(10 + n(-1 - 7k + (-2 + k(2+k))n))} \end{cases}$$
And: $p^* = a - Q^*$.
Second Order Conditions:

$$\begin{cases} \frac{\partial^2 \pi_i}{\partial q_i^2} < 0, \forall i \in out \\ \frac{\partial^2 \pi_M}{\partial q_M^2} < 0 \end{cases}$$
$$\Rightarrow \begin{cases} -2 - \frac{1}{k_i + \frac{k}{n-2}} < 0 \\ -2 - \frac{1}{2k_i - k} < 0 \end{cases}$$

Appendix H : Proof of proposition 7

We have to determine if prices with divestiture to each firm and with divestiture to one firm can be the same. We find that prices are equal if : $k = 0$, $k = \frac{2+n}{n}$ or $k = \frac{a-c}{a+c(e-2)} + \frac{2}{n}$.

$k = \frac{2+n}{n} > 1$ so it is not possible.

Moreover, we compute the value of price with divestiture in case of $k = \frac{a-c}{a+c(e-2)} + \frac{2}{n}$ and we obtain:

$$p^*(k = \frac{a-c}{a+c(e-2)} + \frac{2}{n}) = c$$

So equilibrium profit of the merged entity for $k = \frac{a-c}{a+c(e-2)} + \frac{2}{n}$ would be negative.

Consequently, equilibrium price with divestiture to one firm is never equal to equilibrium price to divestiture to each firm. Then it is straightforward to prove that post-merger price with divestiture to each firm is always lower than post-merger price with divestiture to only one firm.

Appendix I : Graphical representation of e_1, e_2 and e_3

The inflection point on the curve e_2 is for $k = \bar{k}$[12]. The inflection point of the curve e_3 is for $k = \tilde{k}$.

with
$$\tilde{k} = \frac{n(8 - 4n - 8n^2 + 4n^3) - 2\sqrt{2}(n-2)n^{5/2}\sqrt{(n+1)}}{2(2n^2 - 2n^3 - 2n^4 + n^5)}$$

$$\frac{\tilde{k}}{\bar{k}} = \frac{2(n-2)(n+1)(2n-1)}{(-2 + 2n^2 + \sqrt{2}n(3/2)\sqrt{(1+n)}(4 + 4n - \sqrt{6}\sqrt{(n+1)(3+2n)})}$$

$\frac{\tilde{k}}{\bar{k}} = 1$ for $n = 3$ ($n = 3$ means that there is only one outsider firm so divestiture to one firm or divestiture to each firm in the industry is the same). For $n > 3$, $\tilde{k} > \bar{k}$.

$$e_3(\tilde{k}) = \frac{4\sqrt{n}(n+1)(2c(n+1) - a) + \sqrt{2}\sqrt{(n+1)}(-a(3+2n) + c(4 + n(7+6n)))}{4c\sqrt{n}(n+1)(2n+1) + \sqrt{2}c\sqrt{n+1}(1 + n(5+6n))}$$

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[12]see appendix D

$$e_2(\bar{k}) = \frac{3a(-3+n+2n^2) + c(18 + n(21 + 7n + 2n^2)) - 2\sqrt{6}(a-c)n\sqrt{(n+1)(3+2n)}}{(3+n)^2(c+2n)}$$

$$e_3(\tilde{k}) - e_2(\bar{k}) = \frac{2(a-c)(-9 - \sqrt{2}(n-2)\sqrt{n}\sqrt{(n+1)}(3+n)^2 + \sqrt{6}(n-1)^2 n\sqrt{(n+1)(3+2n)} - n(33 + n(-3 + n(2n-9))))}{(n-1)^2(3+n)^2(c+2cn)}$$

$e_3(\tilde{k}) - e_2(\bar{k}) = 0$ for $n = 1$ and $n = 3$. For $n > 3$, $e_3(\tilde{k}) > e_2(\bar{k})$

Moreover $e_3 = e_1$ for $k = \frac{n-2}{n(n-1)}$ and $\frac{n-2}{n(n-1)} \geq \frac{1}{2n}$ for $n \geq 3$.

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