On the Price Effects of Horizontal Mergers: A Theoretical Interpretation*

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Abstract

Horizontal mergers are usually under the scrutiny of antitrust authorities due to their potential undesirable effects on prices and consumer surplus. "Ex-post" evidence, however, suggests that not always these effects take place and even relevant mergers may end up having negligible price effects. The analysis of mergers in the context of non-localized spatial competition can reconcile theory with that evidence: both positive and zero price effects are possible outcomes of the merger activity in this framework.

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1 Introduction

Antitrust authorities worldwide are highly concerned with the price effects of mergers (Whinston, 2007). When two or more firms operating in the same market merge, the concentration of the market increases and this may drive to undesirable increases in prices with a consequent damage for consumers. According to Weinberg (2008), “the agencies review mergers in an effort to identify and block mergers if they would increase prices”. For these reasons several approaches have been developed in the literature and by the practitioners to evaluate the price effects: "event studies" based on the behaviour of the stock markets evaluation of the companies involved, "simulation studies" based on theoretical models of mergers and appropriately parametrized and "direct studies" of the price effects within a specific industry (Pautler, 2001; Weinberg, 2008). Retrospective evidence on the price effects of mergers is then available. A common trait of this literature is that not always anti-competitive effects arise. This may be due to efficiency gains to be exploited and passed through to prices or simply to the fact that price effects are not significantly affecting consumers welfare. For example, Ashenfelter-Hosken (2008) find that in two out of the five mergers they consider no substantial price effects were registered. In particular, the Aurora-Kraft syrup mergers had almost no effect while the Marathon-Ashland joint venture in the gasoline sector had negative (although non-significant) medium term effects on prices. Ashenfelter-Hosken-Weinberg (2009) assert that the evidence gathered at date is not sufficient to conclude on the anti-competitive effects of mergers. In other words, even significant mergers not always have a sensible negative effect on consumers. This paper proposes a simple theoretical analysis of mergers in differentiated product industries which is consistent with the evidence of horizontal mergers producing substantial price effects and others whose effects are instead negligible.

The paper studies the effects of horizontal mergers in the context of "non-localized" competition. The "traditional" approach to spatial competition uses the circular city model of Vickrey (1964), also referred to as the Salop (1979) model: one of its limit is to address only "localized" spatial competition (Rothschild, 2000). Chen-Riordan (2007) develop a new analytical tool to analyze spatial differentiation which naturally fits to the idea of "non-localized" competition, the spokes model. In this model firms are located at the extreme of a market constituted of several spokes all linked at a common centre. There may be more spokes than firms (Chen-Riordan, 2007) or as many spokes as firms (Caminal-Claici, 2007). The model has three main properties. First, it allows multi-firm spatial competition with no neighbouring effects; second, it captures monopolistic competition *à la Chamberlin* as the number of firms and spokes tends to infinity; third, for some regions of the parameters, competition has a price increasing effect: this is due to the higher elasticity of demand in monopolistic segments as compared to the competitive ones. The model is particularly useful to analyze markets in which consumers have a strong preference for a specific brand while being rather indifferent among alternatives: natural examples are markets for a composite good that requires original parts to be completed or repaired. In a merger context, the spokes approach has a desirable feature: competition is non-localized and in equilibrium all firms are competing against each other. The effect of a merger is then to reduce the intensity of competition. The price effect, however, depends on which part of the market firms are targeting when setting the prices. The demand for the firms' product is composed of several segments characterized by different elasticities. There exist, then, equilibria in which price effects are positive and important. Other equilibria display no price effects at all: these correspond to "kink" equilibria of the model (Economides, 1989, 1993). These results can be interpreted as a possible explanation of the mixed evidence on the mergers effects proposed in the empirical literature. Moreover, mergers activity not only damages consumers: increased prices imply also demand effects that lead to increased transportation costs and, consequently, to negative welfare effects.

The contribution of this paper is related to the literature on endogenous mergers. The goal of most papers on the topic is to solve two paradoxes posed by the game theoretical analysis of mergers and coalition formation. The first is the Salant-Switzer-Reynolds (1983) result who report that in a Cournot oligopoly with homogeneous products, linear demand and cost functions, a merger is beneficial for participating firms if more than 80 per cent of all firms merge. The second is Deneckere-Davidson (1985) show that under price competition and differentiated products mergers are always profitable for insiders. However, according to the intuition of Stigler (1950), the equilibrium displays free-riding properties: "outsiders" always earn higher profits than "insiders". This property also characterizes our analysis: in the regions where prices are increasing following a merger, insiders' prices raise more than outsiders' ones. In other instances, however, prices do not increase. Brito (2003) considers mergers in the context of the circular city. He shows that even if market power is the motivation for merger, firms can be interested in being insiders (pre-emptive merger) and the impact of the merger on the rival firms depends on their location. Firms can prefer to be insiders even if some of the outsiders benefit more (but others less) from the merger than insiders. In this context, he finds that mergers have relevant impacts on prices only if one of the neighbouring firms takes part into it. Our zero-price effect result, however, is obtained in a non-localized framework in which proximity of firms plays no relevant role: in the spokes model competition takes place between all firms, no matter their relative market location. Braid (1999) considers two-dimensional competition between firms located on a grid: the price effects of a merger are lower due to a reduction in the benefits of merging for insiders. As opposed to the standard single dimension, this feature implies that the incentives to raise prices are spread in different directions.

Our results contribute to the rapidly flourishing literature adopting the spokes model as a tool for analysis of multi-firm differentiated product competition[1]. The analysis of the model is extended to allow for asymmetric firms. Taking into account asymmetries may be quite complex in other models of product differentiation as the circular city model (Brito, 2003 ; Borla, 2008; Syverson, 2004; Vogel, 2008; Alderighi-Piga, 2008). In our approach the non-localized nature of competition avoids complex feedback effects on prices of the neighbouring firms, making the exercise easier although non-trivial. This property may strenghthen the case for the spokes model as a convenient and reasonable alternative to the circular city in addressing spatial competition between n firms.

The rest of the paper is organized as follows. Section 2 briefly introduces the spokes model. Mergers are introduced in section 3. Section 4, discusses the price effects of mergers, comparing the *pre-merger* with the *post-merger* equilibria. Concluding remarks follow in section 5. All mathematical derivations can be found in Appendices A and B.

2 The Framework

Consider the model introduced by Chen-Riordan (2007). The market has a spatial structure made up of N spokes of constant length, normalized to $1/2$, with a common core. Suppose there are n firms entering the market and $n \leq N$ is exogenously given. Each firm locates at the extreme of its own spoke and supplies an homogeneous good: transportation costs are the only source of differentiation. Customers are uniformly distributed over all the N spokes. The unit transportation cost t is constant and normalized to one and consumers have a homogeneous valuation of the good v.

[1] The spokes model is used, between others, by Caminal-Claici (2007) in the context of loyalty rewarding schemes, by Caminal-Granero (2008) to study the provision of quality by multi-product firms, by Ganuza-Hauk (2005) to address allocation of ideas in tournaments, by Chen-Riordan (2007) on vertical integration.

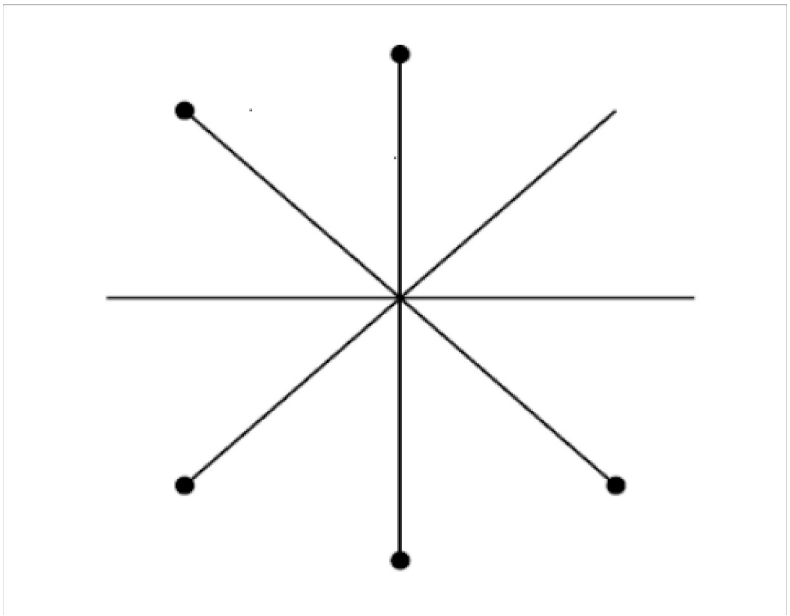

Figure 1: The spokes model, Chen-Riordan (2007), $n = 5$ and $N = 8$.

Tractability requires that consumers only care about the brand located on the same spoke as they are and a finite subset of the $N - 1$ alternative brands: we follow Chen-Riordan (2007) and assume that a randomly picked consumer on a spoke has only one alternative brand. Both the first and the second brand or both may not exist in equilibrium, so the market can be not fully covered. For expositional convenience and without loss of generality the marginal cost is set to equal zero and the profit function of a generic firm i is:

$$\pi_i(p_i, p_{-i}) = p_i D(p_i, p_{-i})$$

From the firm's viewpoint there are different types of customers[2]:

1. Customers on i-th firm's spoke that have one of the remaining firms as

[2] Despite recognizing different types of customers, firms use a unique price and are not allowed any kind of price discrimination. Reggiani (2009) studies spatial price discrimination in the spokes model.

an alternative. The demand from this group is defined by identifying the location $\hat{x}$ of the consumers who are indifferent between buying from i or buying from the rival firm α:

$$\hat{x} = \max\left\{\min\left\{\frac{1 + p_\alpha - p_i}{2}; 1\right\} 0\right\}$$

The constraints imposed simply require the customer to be located on either of the spokes and not outside.

2. Customers on the i-th firm's spoke who do not have an existing alternative brand *and* customers who do not have a first favorite brand but have i as a second favorite. The indifferent customer in the set of these two types is identified by: $\check{x} = \max\left\{\min\left\{v - p_i, 1\right\}, 0\right\}$

The demand function faced by firm i can then be defined as:

$$\begin{aligned} D_i(p_i, p_{-i}) &= \frac{2}{N}\frac{1}{N-1}\sum_{\alpha=1..n}^{\alpha\neq i} \max\left\{\min\left\{\frac{1}{2} + \frac{p_\alpha - p_i}{2}; 1\right\} 0\right\} + \\ &\quad \frac{2}{N}\frac{N-n}{N-1} \max\left\{\min\left\{v - p_i, 1\right\}, 0\right\} \end{aligned}$$

where $2/N$ is the density of consumers at each point of the spokes, $1/(N-1)$ is the probability of j being a customers' second favorite brand and $(N-n)/(N-1)$ is the probability of a consumer having no first or second favorite brand. Also, the following regularity conditions need to be satisfied: $|p_\alpha - p_i| < 1 \ \forall \alpha \neq i$ and $v - p_i \geq 1/2$ to ensure that competition between firms occurs.

The demand function can then be rewritten as:

$$D_i(p_i, p_{-i}) = \begin{cases} \frac{2}{N}\frac{1}{N-1}\sum_{\alpha\neq i}\left(\frac{1}{2} + \frac{p_\alpha - p_i}{2}\right) + \frac{2}{N}\frac{N-n}{N-1} v - p_i & \text{if } \frac{1}{2} \leq v - p_i \leq 1 \\ \\ \frac{2}{N}\frac{1}{N-1}\sum_{\alpha\neq i}\left(\frac{1}{2} + \frac{p_\alpha - p_i}{2}\right) + \frac{2}{N}\frac{N-n}{N-1} & \text{if } \quad v - p_i > 1 \end{cases} \quad (1)$$

The first order conditions identifying the equilibrium prices are given by:

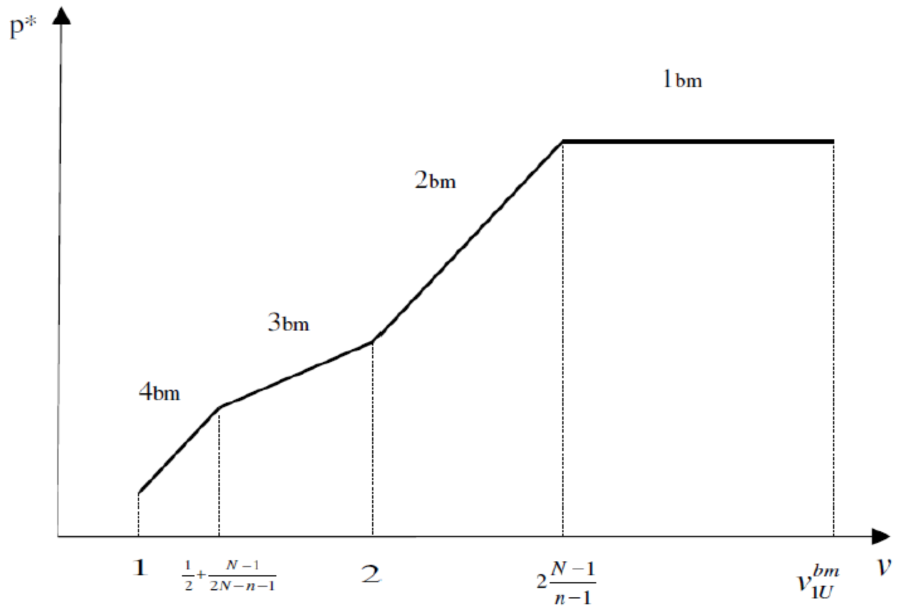

Figure 2: The before merger equilibrium prices, Chen-Riordan (2007).

$$\frac{\partial \pi_i}{\partial p_i} = D_i(p_i, p_{-i}) + p_i \frac{\partial D_i(p_i, p_{-i})}{\partial p_i} = 0$$

Given the definition of the demand and profit functions, it can be checked that there exist four possible equilibrium regions.

The equilibrium regions can be characterized depending on the parameter v, the valuation of the good. As in Chen-Riordan (2007), the equilibrium prices before merger p^*_{bm} are defined as in Table 1.

Table 1. Before Merger Equilibrium Prices

Region	Range	p^*_{bm}
1bm	$v \in [\frac{2(N-1)}{n-1}; v^{bm}_{1U}]$	$\frac{(2N-n-1)}{n-1}$
2bm	$v \in [2; \frac{2(N-1)}{n-1}[$	$v-1$
3bm	$v \in [\frac{N-n-3}{N-n-1}; 2[$	$\frac{2v(N-n)+(n-1)}{4N-3n-1}$
4bm	$v \in [1; \frac{N-n-3}{N-n-1}[$	$v-\frac{1}{2}$

where $v^{bm}_{1U} = 2\frac{N-1}{n-1} + \frac{(2N-n-1)}{2(N-n)}$.The derivation of the equilibrium expressions and the regions in which they hold follow Chen-Riordan (2007) and are reported in Appendix A. As illustrated in Figure 2 the price is a non-decreasing function of the value of the good v. For values above v^{bm}_{1U} a pure strategy equilibrium of the game does not exist. A too large valuation of the good implies firms have a unilateral incentive to raise their price to $p = v-1$ which is however not an equilibrium either.

In Region 1 standard oligopoly competition takes place: v is large enough so that firms focus on the segment of consumers who have both a first and a second favourite brand, in other words the demand that the firm shares with all competing firms. The price is independent of v and depends only on the number of firms and spokes. All consumers who have a preference for an existing brand participate in the market.

Region 2 is characterized by a kink in the demand function: firms concentrate on extracting surplus on the indifferent consumers who do not have a second favourite brand. Also in this case all the consumers with at least one favourite brand are served in equilibrium. The price increases linearly with v: this is the parameter characterizing the indifferent consumers that firms consider when setting the price.

In Region 3 firms' prices are driven by the indifferent consumer who does not have an alternative. All surplus is extracted from this type of consumer and some of them are not served in equilibrium. On the other hand, the indifferent consumer who has both the first and the second brand available maintains a positive surplus. As v increases, price increases as it is possible to extract more surplus from the consumers on the monopolistic segment of the market. This region has the interesting property that price is increasing with the number of active firms, as demand is more elastic in the monopolistic segment.

Region 4 is characterized by a different kink of the demand function: firms focus on the indifferent consumer who has its brand as first favourite. As in Region 3 also in this region not all consumers with at least one favourite brand are served: in particular, only consumers with a first favourite brand participate on the market in this case. For even lower values of v an equilibrium would exist but all firms would be local monopolists serving only part of the consumers located on their spoke.

3 Horizontal Mergers

Following the literature, it is assumed that the merging firms maximize their joint profits. We abstract from bargaining considerations: the after merger profits are split in equal parts between the participating firms. In other words, the only effect of a merger is to create a multi-product firm (the "merged entity"): the merging firms supply their product independently but they adopt joint decisions on their prices to maximize joint profits. These assumptions make our results comparable with Caminal-Granero (2008) who explicitly address the role of multi-product firms in supplying variety. Their analysis focuses on a multi-product firm competing against a fringe of firms in the equivalent of Region 1 described above; in what follows, instead, after merger asymmetric competition is analyzed in all equilibrium regions.

3.1 The Effects of a Merger

Suppose that a merged entity M has been formed by the merger of k of the n symmetric firms of the benchmark model. Denote by $i \in I = \{1, ..., k\}$ a firm belonging to M. All other firms are symmetric and indexed by $i \in$

$O = \{k+1, ..., n\}$. Let denote by S the set of all firms($S = I \cup O$). While the number of spokes N is exogenous and not affected by the merger, the number of active firms reduces to $m = n - k + 1$.
Focus first on the merged firms who constituted M. As these firms keep their price independence, the equation:

$$v - p_i - x = v - p_j - (1 - x) \quad \forall i \in I,\ \forall j \in S, \forall j \neq i$$

still identifies the indifferent customers who have an alternative brand existing on the market and the set of indifferent consumers is described by:

$$\hat{x}_{ij} = \max\left\{\min\left\{\frac{1}{2} + \frac{p_j - p_i}{2}; 1\right\} 0\right\} \quad \forall i \in I,\ \forall j \in S, \forall j \neq i$$

Notice, however, that now there are two subsets of indifferent consumers: consumers whose other brand is supplied by one of the other firms taking part to the merger ($j \in M,\ j \neq i$) *and* consumers whose other brand is supplied by one of the outsiders ($j \in O$). Indifferent customers with no kind of alternative brand are still identified by:

$$\check{x}_i = \max\left\{\min\left\{v - p_i, 1\right\}, 0\right\} \quad \forall i \in I$$

The conclusion is that, from the perspective of one of the firms who took part to the merger and constituted firm M there are three types of customers after the merger:

1. $\frac{k-1}{N-1}$ customers whose second favourite brand is supplied by factories located on other spokes but belonging to M, the firm resulting from the merger;

2. $\frac{n-k}{N-1}$ customers who have an alternative brand not supplied by other factories affiliated to M;

3. $\frac{N-n}{N-1}$ customers who do not have a second favourite brand.

Notice that the merger creates no market expansion effect. The agents that do not have a first or a second favourite brand either on the market are excluded: the fraction of this type of consumers is unaffected by the merging activity.
The demand function of the merged entity M is defined by the segments served by the k firms. Each of the segments is defined as:

$$D_i(p_i, p_{-i}) = \frac{2}{N}\left\{ \frac{1}{N-1}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{k} \max\left\{0, \min\left\{\frac{1}{2}+\frac{p_\alpha - p_i}{2}, 1\right\}\right\} + \right.$$
$$+\frac{1}{N-1}\sum_{\alpha=k+1}^{n} \max\left\{0, \min\left\{\frac{1}{2}+\frac{p_\alpha - p_i}{2}, 1\right\}\right\} +$$
$$\left. +\frac{N-n}{N-1}\min\left\{\max\left\{v-p_i, 0\right\}, 1\right\} \right\} \quad \forall i \in I$$

The first term represents consumers with both favourite brands being supplied by M. The second term consumers whose second favourite brand is supplied by one of the remaining firms. The third term the consumers whose only desired brand is supplied by the firm. The demand of each of this segment is weighted by the respective probabilities of a given consumer being one of the three possible types recalled.

The demand function faced by the outsiders is:

$$D_j(p_j, p_{-j}) = \frac{2}{N}\left\{ \frac{1}{N-1}\sum_{\alpha=1}^{k} \max\left\{0, \min\left\{\frac{1}{2}+\frac{p_\alpha - p_j}{2}, 1\right\}\right\} \right.$$
$$+\frac{1}{N-1}\sum_{\substack{\alpha=k+1\\ \alpha\neq j}}^{n} \max\left\{0, \min\left\{\frac{1}{2}+\frac{p_\alpha - pj}{2}, 1\right\}\right\} +$$
$$\left. +\frac{N-n}{N-1}\min\left\{\max\left\{v-p_j, 0\right\}, 1\right\} \right\} \quad \forall j \in O$$

The three terms represent, respectively, the demand faced from consumers who have as other favourite a brand supplied by firm M, consumers who have as other favourite a brand supplied by another non-merged firm and consumers whose only desired brand is supplied by the firm. Notice that the weighting depends only on the number of spokes occupied and not on the number of active firms: despite the latter decreased after the merger, the former was left unmodified. The profit functions for the merged entity and for each outsider are respectively:

$$\pi_M = \sum_{\alpha=1}^{k} p_\alpha D_\alpha(p_\alpha, p_{-\alpha})$$

$$\pi_j = p_j D_j(p_j, p_{-j}) \;\; \forall j \in O$$

The first order conditions for the merged firms are:

$$\frac{\partial \pi_i}{\partial p_i} = D_i(p_i, p_{-i}) + p_i \frac{\partial D_i(p_i, p_{-i})}{\partial p_i} + \sum_{\substack{\alpha=1 \\ \alpha \neq i}}^{k} p_\alpha \frac{\partial D_\alpha(p_\alpha, p_{-\alpha})}{\partial p_i} = 0 \;\; \forall i \in I$$

where the first derivatives of the demand function are respectively:

$$\frac{\partial D_i(p_i, p_{-i})}{\partial p_i} = \begin{cases} -\frac{n-1}{N(N-1)} - \frac{2(N-n)}{N(N-1)} & \text{if } \; \frac{1}{2} \leq v - p_i \leq 1 \\ -\frac{n-1}{N(N-1)} & \text{if } \;\; v - p_i > 1 \end{cases} , \forall i \in I$$

$$\frac{\partial D_\alpha(p_\alpha, p_{-\alpha})}{\partial p_i} = \frac{1}{N(N-1)} \;\; \forall \alpha \in I, \forall i \in I, \forall \alpha \neq i$$

The first order conditions for the non-merged firms are:

$$\frac{\partial \pi_j}{\partial p_j} = D_j(p_j, p_{-j}) + p_j \frac{\partial D_j(p_j, p_{-j})}{\partial p_j} = 0 \;\; \forall j \in O$$

where the first derivatives of the demand function are:

$$\frac{\partial D_j(p_j, p_{-j})}{\partial p_j} = \begin{cases} -\frac{n-1}{N(N-1)} - \frac{2(N-n)}{N(N-1)} & \text{if } \; \frac{1}{2} \leq v - p_j \leq 1 \\ -\frac{n-1}{N(N-1)} & \text{if } \;\; v - p_j > 1 \end{cases} \;\; \forall j \in O$$

Comparing the first order conditions of the merged and the non-merged firms it is clear that the merger imposes on each firm participating to internalize the externalities imposed by one's own price choices on the demand for other brands. This property, first illustrated by Deneckere-Davidson (1985), plays an important role in determining the results discussed in Section 4.

Imposing a regularity condition with the same role of the one adopted in the benchmark case, i.e. $\frac{|p_\alpha - p_i|}{2} < \frac{1}{2}$ $\forall \alpha \in I, \forall i \in I, \alpha \neq i$, the analysis of the equilibria can be performed. In particular, the analysis of the effects of the merger on prices and profits is proposed in what follows.

3.2 The After-Merger Equilibrium

The results of the analysis proposed can be summarized reporting the after-merger equilibrium prices for both merging and non-merging firms.

The equilibrium prices for the merged firms,denoted by p_m^*, are reported in Table 2.

Table 2. Merged Firms' Equilibrium Prices

Region	Range $(v_{\min}, v_{\max})$	p_m^*
1m	$\frac{(4nN-3n-2N-kn-k^2-2k+1)}{(2n+k-2)(n-k)}; v_{1U}^m$	$\frac{(2n-1)(2N-n-1)}{(n-k)(2n+k-2)}$
2m	$\frac{4N-2n-k-1}{2N-n-k}; \frac{2N-k-1}{n-k}$	$v-1$
3m	$\frac{1}{2}\frac{16N^2-12Nn-4Nk-12N+2n^2+3nk+2k+2}{8N^2-8Nn-4Nk-2N+2n^2+3nk-k^2+2k};$ $\frac{16N^2-16Nn-4Nk-8N+4n^2+3nk+3n-k^2+2k+1}{8N^2-8Nn-4Nk-2N+2n^2+3nk-k^2+2k}$	$\frac{(2vN-2vn+n-1)](4N-2n-1)}{16N^2-20Nn+4Nk+4N+6n^2+3nk+2n-k^2+2k}$
4m	$1; \frac{1}{2}\frac{4N-n-k-2}{2N-n-1}$	$v-\frac{1}{2}$

The equilibrium prices and profits for the non merged firms, denoted by p_{nm}^*, are reported in Table 3.

Table 3. Non-merged Firms' Equilibrium Prices

Region	Range $(v_{\min}, v_{\max})$	p^*_{nm}
1nm	$\frac{(4Nn-2kN-4n-k^2+3k)}{(2n+k-2)(n-k)}; v^{nm}_{1U}$	$\frac{(2N-n-1)(2n-k)}{(n-k)(2n+k-2)}$
2nm	$2; 2\frac{N-1}{n-1}$	$v-1$
3nm	$\frac{16N^2-16Nn-4Nk-8N+4n^2+2nk+4n-k^2+3k}{8N^2-8Nn-2Nk-4N+2n^2+nk+2n-k^2+2k};$ $\frac{1}{2}\frac{16N^2-12Nn-4Nk-12N+2n^2+nk+6n-k^2+4k}{8N^2-8Nn-2Nk-4N+2n^2+nk+2n-k^2+2k}$	$\frac{(2vN-2vn+n-1)(4N-2n-k)}{16N^2-20Nn-4Nk-4N+6n^2+3nk+2n-k^2+2k}$
4nm	$1; \frac{1}{2}\frac{4N-n-3}{2N-n-1}$	$v-\frac{1}{2}$

The after-merger market price p^*_{am} is defined as:

$$p^*_{am} = \frac{k}{n}p^*_m + \frac{n-k}{n}p^*_{nm}$$

All the details on the identification of the after-merger equilibria are reported in Appendix B.

4 Results and Discussion

4.1 The main result

Bringing together the results of the pre-merger benchmark situation in Section 2 and the post-merger equilibria in Section 3, we identify *four equilibrium regions.* These regions can be seen as the analog of the four equilibrium regions analyzed by Chen-Riordan (2007): the values of parameters corresponding to each of the region is given by the appropriate boundary between the ones defining the before merger, the merged and non merged firms equilibria. As we shall see, equilibria in these regions also share the same properties of the ones in Chen-Riordan (2007) and briefly recalled in Section 2. The four equilibrium regions we focus on are defined in terms of the parameter v as:

$$\begin{array}{ll} \text{Region 1} & v_{1D} < v \leq v_{1U} \\ \text{Region 2} & v_{2D} < v \leq v_{2U} \\ \text{Region 3} & v_{3D} < v \leq v_{3U} \\ \text{Region 4} & v_{4D} < v \leq v_{4U} \end{array}$$

where the limiting values identifying each region are defined as follows:

$$v_{1D} = \max\{2\frac{N-1}{n-1}, \frac{(4nN-3n-2N-kn-k^2-2k+1)}{(2n+k-2)(n-k)}, \frac{(4Nn-2kN-4n-k^2+3k)}{(2n+k-2)(n-k)}\}$$

$$v_{2D} = \max\{2, \frac{4N-2n-k-1}{2N-n-k}, 2\}$$

$$v_{3D} = \max\{\frac{4N-n-3}{2(2N-n-1)}, \frac{1}{2}\frac{16N^2-12Nn-4Nk-12N+2n^2+3nk+2k+2}{8N^2-8Nn-4Nk-2N+2n^2+3nk-k^2+2k}, \frac{16N^2-16Nn-4Nk-8N+4n^2+2nk+4n-k^2+3k}{8N^2-8Nn-2Nk-4N+2n^2+nk+2n-k^2+2k}\}$$

$$v_{4D} = \max\{1, 1, 1\}$$

for downwards boundaries, and:

$$v_{1U} = \min\{v_{1U}^{bm}, v_{1U}^{m}, v_{1U}^{nm}\}$$

$$v_{2U} = \min\{2\frac{N-1}{n-1}, \frac{2N-k-1}{n-k}, 2\frac{N-1}{n-1}\}$$

$$v_{3U} = \min\{2, \frac{16N^2-16Nn-4Nk-8N+4n^2+3nk+3n-k^2+2k+1}{8N^2-8Nn-4Nk-2N+2n^2+3nk-k^2+2k}, \frac{1}{2}\frac{16N^2-12Nn-4Nk-12N+2n^2+nk+6n-k^2+4k}{8N^2-8Nn-2Nk-4N+2n^2+nk+2n-k^2+2k}\}$$

$$v_{4U} = \min\{\frac{4N-n-3}{2(2N-n-1)}, \frac{1}{2}\frac{4N-n-k-2}{2N-n-1}, \frac{1}{2}\frac{4N-n-3}{2N-n-1}\}$$

for upwards boundaries.

The analysis proposed focuses on the comparison of p^*_{bm} and p^*_{am} as defined in Section 2 and 3 in the regions defined above. Proposition 1 states the main result of the paper.

Proposition 1 *A merger leads to an increase in the market price in Region 1 and Region 3; it has no price effect in Region 2 and Region 4.*

Proof In Region 1 the pre- and post-merger equilibria are given by:

$$p_{bm}^{*}=\frac{(2N-n-1)}{n-1};\ p_{am}^{*}=\frac{2N-n-1}{n}\frac{2n^{2}+k^{2}-kn-k}{2n^{2}-(k+2)n-k^{2}+2k}$$

so that the difference between the two can be written as:

$$p_{am}^{*}-p_{bm}^{*}=(2n-1)\frac{k}{n}\frac{k-1}{n-1}\frac{2N-n-1}{2n^{2}-(k+2)n-k^{2}+2k}$$

which, as $k<n$, is always positive. Then $p_{am}^{*}>p_{bm}^{*}$. In Region 2 it is immediate to verify that $p_{am}^{*}=p_{bm}^{*}=v-1$. In Region 3, consider p_{m}^{*} and p_{nm}^{*} separately. It turns out that:

$$p_{m}^{*}-p_{bm}^{*}=(k-1)\frac{4N-3n+k-1}{4N-3n-1}\frac{2Nv-2nv+n-1}{16N^{2}-4Nk-20Nn-4N-k^{2}+3kn+2k+6n^{2}+2n}$$

and

$$p_{nm}^{*}-p_{bm}^{*}=\frac{k\,(k-1)}{4N-3n-1}\frac{2Nv-2nv+n-1}{16N^{2}-4Nk-20Nn-4N-k^{2}+3kn+2k+6n^{2}+2n}$$

implying that both p_{m}^{*} and p_{nm}^{*} are higher than p_{bm}^{*}, maintaining that $k<n\leq N$. But if both p_{m}^{*} and p_{nm}^{*} are higher than p_{bm}^{*} then, by definition, also $p_{am}^{*}>p_{bm}^{*}$. In Region 4, $p_{am}^{*}=p_{bm}^{*}=v-\frac{1}{2}$. *Q.E.D.*

Proposition 1 is well illustrated by Figure 3. The picture illustrates the before and after-merger prices in the four equilibrium regions. As it can be seen, in Region 1 and 3 the after-merger market price dominates the before merger price; in Region 2 and 4 the two prices coincide.

4.2 Discussion

Proposition 1 states that in two of the equilibrium regions prices increase as a consequence of the merger while this is not the case for the remaining two

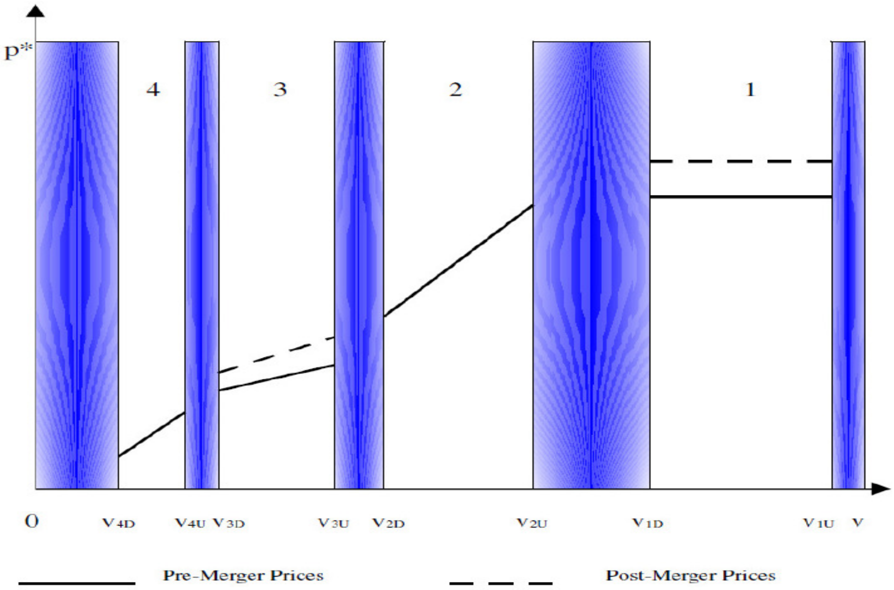

Figure 3: The after merger equilibrium prices.

regions. We shall first describe the mechanisms that lead to this result before providing an interpretation of those.

In Region 1, "standard" oligopolistic competition takes place. Under these conditions the mechanisms highlighted by Deneckere-Davidson (1985) are in operation: as the best response functions are upwards sloping, both insiders and outsiders raise their prices, earning higher profits. The intuition provided by Deneckere-Davidson (1985) is the following. A given outsider faces competition from both the merged entity and the other outsiders. Then, it shares with a given insider $n-2$ competitors. But they both face another competitor. For the outsider firm this competitor is a member of the merged entity, so a firm charging a higher price. The insider, on the other hand, faces competition of another outsider firm, which is charging a lower price. This implies that outsiders face less fierce competition and their profits dominate the ones of insiders. This is exactly what happens in Region 1, creating an overall increase of prices as compared with the benchmark situation.

In Region 2 a "kinked equilibrium" takes place as firms concentrate on extracting surplus on consumers who do not have a second favourite brand and are indifferent between buying or not. For this reason, the merger does not affect prices and profits. The same amount of consumers takes part to the market and prices are unaffected.

Region 3 features a price increase like in Region 1: the reason, however, may seem less intuitive in this case. Competition in this region implies extracting all surplus from the consumers who lack a second alternative brand. In this region the elasticity of demand is larger on the monopolistic segment implying price increasing competition. However, also in this case the mechanisms highlighted by Deneckere-Davidson (1985) are in operation. The best response functions are still upward sloping so that prices of both insiders and outsiders increase, leading to an increase of prices as compared to the benchmark situation.

In Region 4 the "kinked equilibrium" is of a different type: firms focus their attention on the indifferent consumers who have its brand as a first choice. As in Region 3 also in this region not all consumers with at least one favourite brand are served. But as in Region 2 the kinked nature of the equilibrium implies the prices remain unchanged even when a merger takes place.

The analysis of the four equilibrium regions has shown how several economic mechanisms operate in the spokes model. These mechanisms determine the price effects of an horizontal merger between firms. The results can be interpreted as follows: when genuine price competition is in operation, as in Region 1 and 3, then the "free-riding" property of the equilibrium takes place and the classical price increase result highlighted in the existing literature is confirmed. Furthermore, price effects also imply a negative welfare impact. Asymmetric price increases are reflected on different equilibrium demand shares for different firms: this implies an overall increase in transportation costs, impacting negatively on overall welfare. "Kinked equilibria", however, take place in Region 2 and 4 and the equilibrium prices are independent of whether a subset of firms merge. In other words, when firms target a specific key segment of the market mergers do not have any price effect. The results, then, provide the policy maker with an important message. If a market is characterized by product differentiation and firms compete in prices, then a merger may not have detrimental effects for consumers and welfare. Instead, it was shown that for a non-negligible subset of the parameter space, mergers do not have any price effect at all. The model, in other words, seems to reconcile theory with the "ex-post" empirical evidence that not all mergers have important price effects.

The regions just analyzed may seem to capture a rather peculiar case. However, this is not completely true. First of all, their relevance is witnessed by the extent of the sub-space of parameters for which such equilibria take place: this is not sensibly different with respect to the remaining two regions. Moreover in Region 4 firms focus on the indifferent consumer who does not have an alternative so that they only serve their own spoke; however, in Region 2, firms serve all consumers but the ones who have preferences for non-existing brands only. In this sense competition between firms is fully in operation. Finally, it is often observed in the business world that firms target a specific class of indifferent consumers: this might imply that, "*mutatis mutandis*", these results may be relevant to interpret the strategic behaviour of firms in a wide set of situations.

The four regions discussed do not of course exhaust the space of parameters; further regions can be identified as the shadowed areas in Figure 3. We do not present the analysis for the remaining combinations of the parameters, i.e. the blue shaded regions in Figure 3. The results in those regions are quantitatively different from what we presented in this paper; qualitatively, however, they are similar and do not add to our understanding of the price effects and the interpretation we provide. In those regions, in fact, a price increase takes place following a merger in a similar fashion and for the same reasons as in Region 1 and Region 3 presented above.

5 Conclusions

Antitrust authorities focus much of their attention on the price effects of mergers: when two or more firms operating in the same market merge, the concentration of the market increases and this may drive to undesirable price increases. This paper provides a simple theoretical interpretation of the price effects of horizontal mergers which seems consistent with the empirical evidence: many mergers may have an important impact on prices but cases of negligible price effects are not rare either. The results are provided through the analysis of horizontal mergers in a context of spatial but non-localized competition. Two key features of the spokes model, a recently introduced tool to address non-localized competition, are that proximity between firms plays no role and that not all spokes may feature a firm located on it. These properties allow to identify four types of equilibrium with different features. In two of the equilibria, sensible price effects take place as the "free riding" mechanism described by Deneckere-Davidson (1985) is in operation. The merger modifies firms' reaction functions and drives both insiders and outsiders to raise their prices, with consequent harm to consumers and overall welfare. The latest effect operates through increased transportation costs due to the asymmetric demand effects of price increases. The two remaining equilibria, however, have different properties: in those, firms focus only on a specific type of consumer. Mergers then have no effects on prices, which are simply determined by the kink of the demand function that is targeted. These results, then, suggest that a merger will not necessarily imply a sharp increase in the price level: whether this is the case or not will actually depend on the type of consumers that firms are targeting when setting their prices. Consumers may actually benefit from a merger in case this implies cost synergies or other types of efficiency gains.

Besides the intuitive appeal of the results proposed, focusing on the spokes model may also provide a rather realistic interpretation of mergers in several markets. The system of preferences on which the model is built approximates the structure of both geographical markets and product characteristics markets. Examples of the former are geographical structures featuring a centre around which firms concentrate, as in a city with industrial districts at its periphery; of the latter, markets in which brand loyalty is extremely important, as in automotive supplies: when substituting a component of a car, the owner has a preference for the original part and is usually rather indifferent between all other options or a subset of them. The prediction of the model is that the price effect of mergers in such sectors are crucially related to the segment of demand firms focus on when it comes to set prices.

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A Appendix: the Benchmark Case

This appendix follows Chen-Riordan[9] to derive the price equilibrium in the four regions of benchmark case.

Under symmetry, the demand function can be expressed as follows:

$$D_i(p_i, p^*) = \begin{cases} \frac{2}{N}\frac{n-1}{N-1}\left(\frac{1}{2}+\frac{p^*-p_i}{2}\right)+\frac{2}{N}\frac{N-n}{N-1}v-p_i & \text{if } \frac{1}{2}\leq v-p_i\leq 1 \\ \\ \frac{2}{N}\frac{n-1}{N-1}\left(\frac{1}{2}+\frac{p^*-p_i}{2}\right)+\frac{2}{N}\frac{N-n}{N-1} & \text{if } \quad v-p_i>1 \end{cases}$$

and the first derivative of the demand function is:

$$\frac{\partial D_i(p_i, p^*)}{\partial p_i} = \begin{cases} -\frac{2(N-n)+(n-1)}{N(N-1)} & \text{if } \frac{1}{2}\leq v-p_i\leq 1 \\ \\ -\frac{n-1}{N(N-1)} & \text{if } \quad v-p_i>1 \end{cases}$$

Region 1

Assume that: $v-p^*>1$. The first order condition is:

$$\left\{\frac{n-1}{N(N-1)}\left[1+p^*-p_i\right]+\frac{N-n}{N-1}\frac{2}{N}\right\}-p_i\frac{(n-1)}{N(N-1)}=0$$

The best response function is:

$$p_i=\frac{1}{2}p^*+\frac{2N-n-1}{2(n-1)}$$

Imposing symmetry, the equilibrium price turns out to be:

$$p^*=\frac{2N-n-1}{n-1}$$

consistent with the results of Chen-Riordan[9]. Clearly, the equilibrium quantity and profits are, respectively:

$$q^*=\frac{2N-n-1}{N(N-1)} \quad \pi^*=\frac{(2N-n-1)^2}{(n-1)N(N-1)}$$

Incentives to deviate globally from the candidate equilibrium must be checked. The only potentially profitable deviation is to raise price to $p = v - 1$. This would ensure the firm a profit:

$$\pi(v-1) = (v-1)\left(\frac{2}{N}\frac{N-n}{N-1}\right)$$

Such a deviation is not profitable if $\pi \leq \pi^*$ which implies:

$$v \leq \left[2\frac{N-1}{n-1} + \frac{(2N-n-1)}{2(N-n)}\right] = v_{1U}^{bm}(N,n)$$

Region 2

Suppose: $p^* = v - 1$: to show that the candidate price is an equilibrium, it has to be checked that a small increase or decrease in price does not lead to an increase in profit. A small increase in price leads to Region III. The incentive to deviate is then evaluated as:

$$\frac{2}{N}\left[\frac{n-1}{N-1}\frac{1}{2} + \frac{N-n}{N-1}\right] - (v-1)\left[\frac{2(N-n)+(n-1)}{N(N-1)}\right]$$

which implies:

$$v \geq 2$$

A small decrease in price leads to Region I. The incentive to deviate is then evaluated as:

$$\frac{2}{N}\left[\frac{n-1}{N-1}\frac{1}{2} + \frac{N-n}{N-1}\right] - (v-1)\left[\frac{n-1}{N(N-1)}\right]$$

which implies:

$$v \leq 2\frac{N-1}{n-1}$$

Region 3

Assume that: $\frac{1}{2} < v - p^* < 1$. The first order condition is given by:

$$\left\{\frac{n-1}{2\,(N-1)}\left[1+p^*-p_i\right] + \frac{N-n}{(N-1)}v - p_i\right\} - (p_i)\left[\frac{2(N-n)+(n-1)}{(N-1)}\right] = 0$$

which implies a best response function:

$$p_i = \frac{1}{2} + \frac{(n-1)(p^*+1) + 2v(N-n)}{2v(2N-n-1)}$$

and, imposing symmetry, the equilibrium price:

$$p^* = \frac{2v(N-n) + (n-1)}{4N-3n-1}$$

Clearly, the equilibrium quantity and profits are, respectively:

$$q^* = \frac{(2N-n-1)[2v(N-n)-(n-1)]}{N(N-1)(4N-3n-1)}$$

$$\pi^* = \frac{(2N-n-1)[2v(N-n)-(n-1)]^2}{N(N-1)(4N-3n-1)^2}$$

Region 4

Assume $p^* = v - \frac{1}{2}$: to show that the candidate price is an equilibrium, it has to be checked that a small increase or decrease in price does not lead to an increase in profit. A small increase in price leads to an equilibrium region in which the market is not fully covered and firms behave like monopolists. The incentive to deviate is then evaluated as:

$$\frac{2}{N}(v-p^*) - p^* \left[\frac{2}{N}\right]$$

which, around $p^* = v - \frac{1}{2}$, is:

$$\frac{1}{N} - (v - \frac{1}{2})\frac{2}{N}$$

implying:

$$v \geq 1$$

A small decrease in price leads to Region III. The incentive to deviate is then evaluated as:

$$\frac{2}{N}\left[\frac{n-1}{N-1}\frac{1}{2} + \frac{N-n}{N-1}v - \frac{1}{2}\right] - (v-\frac{1}{2})\left[\frac{2(N-n)+(n-1)}{N(N-1)}\right]$$

which implies:

$$v \leq \frac{1}{2}\frac{4N-n-3}{2N-n-1}$$

B Appendix: Identification of the Equilibrium Regions

The identification of the equilibrium regions passes through the characterization of the incentives to deviate of firms from the candidate equilibrium prices. This allows to characterize the threshold values of v reported below and in the text. Let denote by v_{lD} and v_{lU} the values of the $v_{\min}$ and $v_{\max}$ for the region $l, l \in \{1,2,3,4\}$. These values are computed for the merging firms (v_m) and the non merging firms (v_{nm}).

$$\begin{array}{rr} v_{1D} = \max\{\frac{2(N-1)}{n-1}, v_{1D}^{m}, v_{1D}^{nm}\} & v_{1U} = \min\{v_{1U}^{bm}, v_{1U}^{m}, v_{1U}^{nm}\} \\ v_{2D} = \max\{2, v_{2D}^{m}, v_{2D}^{nm}\} & v_{2U} = \min\{2\frac{N-1}{n-1}, v_{2U}^{m}, v_{2U}^{nm}\} \\ v_{3D} = \max\{\frac{1}{2}\frac{4N-n-3}{2N-n-1}, v_{3D}^{m} v_{3D}^{nm}\} & v_{3U} = \min\{2, v_{3U}^{m}, v_{3U}^{nm}\} \\ v_{4D} = \max\{1, v_{4D}^{m}, v_{4D}^{nm}\} & v_{4U} = \min\{\frac{1}{2}\frac{4N-n-3}{2N-n-1}, v_{4U}^{m}, v_{4U}^{nm}\} \end{array}$$

Region 1

In this case parameters are such that $v - p_{bm}^{*} > 1$, $v - p_{m}^{*} > 1$ and $v - p_{nm}^{*} > 1$. In order to check this is an equilibrium, it has to be shown that both merged and non-merged firms do not have an incentive to deviate to a different price. Focus on one of the merged firms, say $i, i \in I$. First, it must not have incentives to deviate to a higher price as $p_i = v - 1$ given the price of the other merging and the non-merging firms is unchanged. This is equivalent to impose the following:

$$\frac{\partial \pi_i}{\partial p_i} \mid_{p_i = v-1, p_m^*,\, p_{nm}^*} = p_i \frac{\partial D_i}{\partial p_i} + D_i + (k-1) p_m^* \frac{\partial D_m}{\partial p_i} \mid_{p_i = v-1, p_m^*,\, p_{nm}^*} \leq 0, \forall i \in I$$

Solving for v drives to the conclusion that it holds if $v \geq v_{1D}^{m}$ where:

$$v_{1D}^{m} = \frac{(4nN - 3n - 2N - kn - k^2 - 2k + 1)}{(2n + k - 2)(n - k)}$$

Moreover, in order for an equilibrium to exist for the merged firms, following Chen-Riordan, it has to be imposed that:

$$\pi_i^* \geq \pi_i^D, \forall i \in I$$

where π_i^D is:

$$\pi_i^D = \frac{2(v-1)D_i(p_m^* + 1, p_{nm}^*)}{N}, \forall i \in I$$

Solving for v, it is found that this condition holds for $v \leq v_{1U}^m(N, n, k)$. It follows that the equilibrium for merging firms exist for:

$$v_{1D}^m \leq v \leq v_{1U}^m$$

Turn now to non-merging firms. If one of the non-merged firms, say j ($j \in O$), tries to deviate it will raise price to $p_j = v - 1$

$$\frac{\partial \pi_j}{\partial p_j} \mid_{p_m^*, p_{nm}^*, p_j = v-1} = p_j \frac{\partial D_j}{\partial p_j} + D_j \mid_{p_m^*, p_{nm}^*, p_j = v-1} \leq 0, \forall j \in O$$

Solving for v drives to the conclusion that it holds if $v \geq v_{1D}^{nm}$ where:

$$v_{1D}^{nm} = \frac{(4Nn - 2kN - 4n - k^2 + 3k)}{(2n + k - 2)(n - k)}$$

Moreover, in order for an equilibrium to exist for the merged firms, following Chen-Riordan, it has to be imposed that:

$$\pi_j^* \geq \pi_j^D, \ \forall j \in O$$

where π_j^D is:

$$\pi_j^D = \frac{2(v-1)D_j(p_{nm}^* + 1, p_m^*)}{N}, \forall \ j \in O$$

Solving for v, it is found that this condition holds for $v \leq v_{1U}^{nm}(N, n, k)$. It follows that the equilibrium for non-merging firms exist for:

$$v_{1D}^{nm} \leq v \leq v_{1U}^{nm}$$

From the benchmark case, it is possible to recall that equilibrium in Region 1 was defined for:

$$\frac{2(N-1)}{n-1} < v \leq v_{1U}^{bm}(N, n)$$

Bringing together all the information, it is verified that an equilibrium which is incentive compatible before and after merger and for both merged and non-merged firms exists for the following subset of values of v:

$$v_{1D} = \max\{\frac{2(N-1)}{n-1}, v_{1D}^{m}, v_{1D}^{nm}\} \leq v \leq \min\{v_{1U}^{bm}, v_{1U}^{m}, v_{1U}^{nm}\} = v_{1U}$$

Region 2

In this case the parameters are such that: $v - p_{bm}^{*} = 1$, $v - p_{m}^{*} = 1$ and $v - p_{nm}^{*} = 1$. Focus first on a given merging firm, say i ($i \in I$). It must be ruled out that she has an incentive to raise her price to $p_i > v-1$ or decrease it to $p_i < v-1$. Consider a price increase, in that case the demand faced by the firm is as if she was in Region 3, given the other firms stick to their equilibrium prices. This is not profitable if:

$$\frac{\partial \pi_i}{\partial p_i} \mid_{p_i > v-1, p_m^*,\ p_{nm}^*} = p_i \frac{\partial D_i}{\partial p_i} + D_i + (k-1) p_m^* \frac{\partial D_m}{\partial p_i} \mid_{p_i > v-1, p_m^*,\ p_{nm}^*} \leq 0, \forall i \in I$$

which implies:

$$v \geq v_{2D}^{m} = \frac{4N - 2n - k - 1}{2N - n - k}$$

Consider instead a price decrease, in that case the demand faced by firm i is as if they were in Region 1, given the other firms stick to their equilibrium prices. This is not profitable if:

$$\frac{\partial \pi_i}{\partial p_i} \mid_{p_i < v-1, p_m^*,\ p_{nm}^*} = p_i \frac{\partial D_i}{\partial p_i} + D_i + (k-1) p_m^* \frac{\partial D_m}{\partial p_i} \mid_{p_i < v-1, p_m^*,\ p_{nm}^*} \leq 0, \forall i \in I$$

implying:

$$v \leq v_{2U}^{m} = \frac{2N - k - 1}{n - k}$$

Turn now to non-merging firms. An analogous reasoning allows to rule out possible deviations. Suppose in particular that firm j raises her price to $p_j > v - 1$. In that case the demand faced by the firm is as if she was in Region 3. In order for this not to be profitable it must be:

$$\frac{\partial \pi_j}{\partial p_j} \mid_{p_m^*, p_{nm}^*, p_j > v-1} = p_j \frac{\partial D_j}{\partial p_j} + D_j \mid_{p_m^*, p_{nm}^*, p_j > v-1} \leq 0, \forall j \in O$$

which in turn implies:

$$v \geq v_{2D}^{nm} = 2$$

If firm j decreases her price, instead, to $p_j < v - 1$. In that case the demand faced by the firm is as if she was in Region 1. In order for this not to be profitable it must be:

$$\frac{\partial \pi_j}{\partial p_j} \mid_{p_m^*, p_{nm}^*, p_j < v-1} = p_j \frac{\partial D_j}{\partial p_j} + D_j \mid_{p_m^*, p_{nm}^*, p_j < v-1} \leq 0, \forall j \in O$$

implying:

$$v \leq v_{2U}^{nm} = 2\frac{N-1}{n-1}$$

From the benchmark case, it is possible to recall that equilibrium in Region 2 was defined for:

$$2 < v \leq 2\frac{N-1}{n-1}$$

Bringing together all the information, it is verified that an equilibrium which is incentive compatible before and after merger and for both merged and non-merged firms exists for the following subset of values of v:

$$v_{2D} = \max\{2, v_{2D}^{m}, v_{2D}^{nm}\} \leq v \leq \min\{2\frac{N-1}{n-1}, v_{2U}^{m}, v_{2U}^{nm}\} = v_{2U}$$

Region 3

In this case the parameters are such that: $\frac{1}{2} < v - p_{bm}^* < 1$, $\frac{1}{2} < v - p_m^* < 1$ and $\frac{1}{2} < v - p_{nm}^* < 1$. It has to be checked that at the proposed equilibrium prices there are no incentives to deviate. In this case there are two possible deviations possibilities for the merged firms and for the non-merged: they can potentially deviate either to $p = v - 1$ or to $p = v - \frac{1}{2}$.

Consider first one of the merging firms, say i ($i \in I$). Suppose she raises the price to $p_i = v - \frac{1}{2}$. For the deviation not to be profitable the following must be imposed:

$$\frac{\partial \pi_i}{\partial p_i} \mid_{p_i = v - \frac{1}{2}, p_m^*, p_{nm}^*} = p_i \frac{\partial D_i}{\partial p_i} + D_i + (k-1) p_m^* \frac{\partial D_m}{\partial p_i} \mid_{p_i = v - \frac{1}{2}, p_m^*, p_{nm}^*} \leq 0, \forall i \in I$$

Solving for v drives to the conclusion that it holds if $v \geq v_{3D}^{m}$ where:

$$v_{3D}^{m} = \frac{1}{2}\frac{16N^2 - 12Nn - 4kN - 12N + 2n^2 + 3nk + 2k + 2}{8N^2 - 8Nn - 4Nk - 2N + 2n^2 + 3k - k^2 + 2k}$$

On the other hand, if the firm decreases her price to $p_m = v - 1$ then it has to be imposed that:

$$\frac{\partial \pi_i}{\partial p} |_{p_i = v-1,\ p_m^*, p_{nm}^*} = p_1 \frac{\partial D_1}{\partial p_1} + D_1 + p_2 \frac{\partial D_2}{\partial p_2} + D_2 |_{p_i = v-1, p_m^*,\ p_{nm}^*} \geq 0, \forall i \in I$$

Solving for v drives to the conclusion that it holds if $v \leq v_{3U}^{m}$ where:

$$v_{3U}^{m} = \frac{16N^2 - 16Nn - 4Nk - 8N + 4n^2 + 3kn + 3n - k^2 + 2k + 1}{8N^2 - 8Nn - 4Nk - 2N + 2n^2 + 3kn - k^2 + 2k}$$

It follows that the equilibrium for merging firms exist for:

$$v_{3D}^{m} \leq v \leq v_{3U}^{m}$$

Turn now to non-merging firms. If one of the non-merged firms, say j ($j \in O$), tries to deviate it will cut her price to $p_j = v - 1$ then for the deviation not to be profitable it must be:

$$\frac{\partial \pi_j}{\partial p_j} |_{p_m^*, p_{nm}^*, p_j = v-1} = p_j \frac{\partial D_j}{\partial p_j} + D_j |_{p_m^*, p_{nm}^*, p_j = v-1} \geq 0, \forall j \in O$$

Solving for v drives to the conclusion that it holds if $v \leq v_{3U}^{nm}$ where:

$$v_{3U}^{nm} = \frac{1}{2}\frac{16N^2 - 12Nn - 4Nk - 12N + 2n^2 + nk + 6n - k^2 + 4k}{8N^2 - 8Nn - 2Nk - 4N + 2n^2 + nk + 2n - k^2 + 2k}$$

On the other hand, if j raises her price to $p_j = v - \frac{1}{2}$ then the following has to be imposed:

$$\frac{\partial \pi_j}{\partial p_j} |_{p_m^*, p_{nm}^*, p_j = v-\frac{1}{2}} = p_j \frac{\partial D_j}{\partial p_j} + D_j |_{p_m^*, p_{nm}^*, p_j = v-\frac{1}{2}} \leq 0, \forall j \in O$$

Solving for v drives to the conclusion that it holds if $v \geq v_{3D}^{nm}$ where:

$$v_{3D}^{nm} = \frac{16N^2 - 16Nn - 4Nk - 8N + 4n^2 + 2nk + 4n - k^2 + 3k}{8N^2 - 8Nn - 2Nk - 4N + 2n^2 + nk + 2n - k^2 + 2k}$$

It follows that the equilibrium for non-merging firms exist for:

$$v_{3D}^{nm} \leq v \leq v_{3U}^{nm}$$

From the benchmark case, it is possible to recall that equilibrium in Region 1 was defined for:

$$\frac{1}{2}\frac{4N - n - 3}{2N - n - 1} < v \leq 2$$

Bringing together all the information, it is verified that an equilibrium which is incentive compatible before and after merger and for both merged and non-merged firms exists for the following subset of values of v:

$$v_{3D} = \max\{\frac{1}{2}\frac{4N - n - 3}{2N - n - 1}, v_{3D}^{m}, v_{3D}^{nm}\} \leq v \leq \min\{2, v_{3U}^{m}, v_{3U}^{nm}\} = v_{3U}$$

Region 4

In this case the parameters are such that:$v - p_{bm}^* = \frac{1}{2}$, $v - p_m^* = \frac{1}{2}$ and $v - p_{nm}^* = \frac{1}{2}$. Focus first on a given merging firm, say i $(i \in I)$. It must be ruled out that she has an incentive to raise her price to $p_i > v - \frac{1}{2}$ or decrease it to $p_i < v - \frac{1}{2}$. Consider a price increase, in that case the demand faced by the firm is as if she was a local monopolist, given the other firms stick to their equilibrium prices. This is not profitable if:

$$\frac{\partial \pi_i}{\partial p_i} \mid_{p_i > v - \frac{1}{2}, p_m^*,\, p_{nm}^*} = p_i \frac{\partial D_i}{\partial p_i} + D_i + (k-1) p_m^* \frac{\partial D_m}{\partial p_i} \mid_{p_i > v - \frac{1}{2}, p_m^*,\, p_{nm}^*} \leq 0, \forall i \in I$$

which implies:

$$v \geq v_{4D}^{m} = 1$$

Consider instead a price decrease, in that case the demand faced by firm i is as if they were in Region III, given the other firms stick to their equilibrium prices. This is not profitable if:

$$\frac{\partial \pi_i}{\partial p_i} \mid_{p_i > v - \frac{1}{2}, p_m^*,\ p_{nm}^*} = p_i \frac{\partial D_i}{\partial p_i} + D_i + (k-1) p_m^* \frac{\partial D_m}{\partial p_i} \mid_{p_i < v - \frac{1t}{2}, p_m^*,\ p_{nm}^*} \leq 0, \forall i \in I$$

implying:

$$v \leq v_{4U}^m = \frac{1}{2} \frac{4N - n - k - 2}{2N - n - 1}$$

Turn now to non-merging firms. An analogous reasoning allows to rule out possible deviations. Suppose in particular that firm j ($j \in O$) raises her price to $p_j > v - \frac{1}{2}$. In that case the demand faced by the firm is the one of a local monopolist. In order for this not to be profitable it must be:

$$\frac{\partial \pi_j}{\partial p_j} \mid_{p_m^*, p_{nm}^*, p_j > v - \frac{1}{2}} = p_j \frac{\partial D_j}{\partial p_j} + D_j \mid_{p_m^*, p_{nm}^*, p_j > v - \frac{1}{2}} \leq 0, \forall j \in O$$

which in turn implies:

$$v \geq v_{4D}^{nm} = 1$$

If firm j decreases her price, instead, to $p_j < v - \frac{1}{2}$. In that case the demand faced by the firm is as if she was in Region 3. In order for this not to be profitable it must be:

$$\frac{\partial \pi_j}{\partial p_j} \mid_{p_m^*, p_{nm}^*, p_j < v - \frac{1}{2}} = p_j \frac{\partial D_j}{\partial p_j} + D_j \mid_{p_m^*, p_{nm}^*, p_j < v - \frac{1}{2}} \leq 0, \forall j \in O$$

implying:

$$v \leq v_{4U}^{nm} = \frac{1}{2} \frac{4N - n - 3}{2N - n - 1}$$

From the benchmark case, it is possible to recall that equilibrium in Region 2 was defined for:

$$1 < v \leq \frac{1}{2} \frac{4N - n - 3}{2N - n - 1}$$

Bringing together all the information, it is verified that an equilibrium which is incentive compatible before and after merger and for both merged and non-merged firms exists for the following subset of values of v:

$$v_{4D} = \max\{1, v_{4D}^m, v_{4D}^{nm}\} \leq v \leq \min\{\frac{1}{2} \frac{4N - n - 3}{2N - n - 1}, v_{4U}^m, v_{4U}^{nm}\} = v_{4U}$$