In Search of Complementarity (from Edgeworth to Slutsky)

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Abstract

The article deals with the early story of the concept of complementarity in consumption, as it developed from the beginning of the marginalist revolution up to Slutsky's famous article. It is aimed at scrutinizing the birth and the development of modern demand theory. In this respect, Slutsky's 1915 article is seen as the culmination of the debates on utility, demand behavior and psychology that would prove fundamental for the shaping of modern demand theory. The works of Edgeworth, Marshall, Auspitz and Lieben, Pareto, Fisher and Slutsky are put in perspectives. The process of de-homogeneizing the concept of complementarity helps to identify different perspectives on demand theory and to make much more precise both the analytical and conceptual contributions of each author. The main thesis of the article are 1) that complementarity was developed as a tool – be it even partial – for studying demand and not as the result of the search for an ordinal definition, and 2) that Slutsky's contribution in this respect was primarily oriented in delineating the common frontiers of economics and psychology.

Introduction

It is well known that Slutsky's 1915 paper, when it was discovered in the thirties, came to be used by Schultz (1935 and 1938) and Hicks (1937 and 1939) to support an ordinal definition of competitive and complementary goods in consumer theory. Notably, it was shown that Slutsky's famous equation could lead to a definition tantamount to the Hicks-Allen definition (see Chipman and Lenfant, 2005).

From this point of view, Slutsky (1915) is interpreted as a starting point for modern utility and demand theory. Curiously enough, Slutsky's article is at odds with such a reassuring story. Indeed, Slutsky himself seems to be puzzled about some properties of the new theory of utility, and more peculiarly he would question the apparent gap between the traditional utility-based definition of complementarity and the ordinal use of utility, without seeing that his own equation offered a new one deprived of the flaws of the other but intuitively less appealing.

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It is enough, I think, to have another look at Slutsky's contribution. Instead of taking Slutsky for someone at the vanguard in economic analysis – who he certainly was –, let us try just for a moment to take him for someone of his own time: someone plunged into the methodological debates of the period. Among many other mathematical economists, Slutsky was interested in complementarity.

The aim of the article is to provide a panorama of the reflections and ideas about complementarity in consumption until Slutsky (1915), and to bring out the meaning of the search for complementarity in different analytical and methodological contexts. Broadly speaking, I will take the "marginalist revolution" as a starting point and take Slutsky (1915) as an analytical breaking point..

It must be clear from the outset that reflections on complementarity at the end of nineteenth century are embedded in widely different uses: either as a support for demand theory (the law of demand and of joint demand), or as a tool to improve the concept of consumer surplus, or else as a more general instrument for price theory. From Jevons (1871) onwards, it would certainly be difficult to find a writer interested in price and demand who does not devote some passages to the notions of substitutes and complements. The only problem is that this fact leads to the much more problematic question: what is it to say that goods are substitutes or complements? What kind of representation would be useful or adequate? Is substitutability accounting for taste or needs? Does it mean something for actual behavior or for intrinsic pleasure? Or, after all, is it just a word, and what ever can it be deserved to? How and to what extent should microeconomic theory allow for positive results relating introspective properties of complementarity and observable properties of demand relations?

Moreover, all those questions about complementarity have certainly harbored what has been called the "ordinalist revolution", at least to the same extent that purely methodological questions did. I mean that the many different ways to tackle the question of substitutes and complements in consumption were certainly a kind of guiding track for testing and evaluating the developments of utility and demand theory. It was a case for applying and revealing methodological principles.

The main thesis that will be developed hereafter is that the analysis of complementarity in many directions during that period, without intensive consideration for homogeneity and compatibility. The derivative thesis is that complementarity was evolved in the upheaval of ordinalism and that it is a truly sensitive barometer of methodological positions of the main protagonists.[2]

[2] Many studies have settled the historical and methodological stakes of complementarity in consumer's choice theory. The present paper departs from what has been written so far by focusing on the methodological underpinnings of the search for complementarity, and by supposing that the so-called ordinalist revolution is not a linear transformation, but instead that it is crossed by many questions: on the role of introspection, on the empirical content of individual preferences, on the practical use of choice theory for demand theory. Schultz (1938) and Stigler (1950 (1965)) provide an historical treatment biased by the idea of an ordinal-empirical development. Samuelson (1974) provides a comparison of old definitions

It is remarkable that works on complementarity are put in different analytical contexts. They are based sometimes on utility, and some other times they are purely geometrical, build on preferences and indifference curves. One can find also many considerations that have a more pedestrian aspect, dealing directly with observational properties of complementarity.[3] Of course, there are many crossovers between those different representations of complementarity. Moreover, all those developments are also immersed – sometimes but not always – within considerations regarding the intrinsic nature of complementarity between some goods and within considerations concerning wants and needs. All this sketches a plentiful landscape that I would like to set to rights.

What is the place of Slutsky in this skein? The most striking fact, indeed, is that he does not seem to attach as much interest in complementarity as all the other writers before and after him! Nevertheless, it is well known that he contributed to a complete reorganization of the line of thought about utility, preferences, and demand. And also that he attracted attention to the rejection of the traditional definition of complements and substitutes.[4] I want to picture Slutsky's article as containing both an analytical and methodological restatement of the relations between utility and demand. Firstly, Slutsky puts forward a mathematical framework in which geometrical representations will play only a secondary role. Secondly, Slutsky destroys the idea of a clear and simple definition of complementarity on the basis of a utility function. Thirdly, he links all his results with a methodological reflection, based on the need for statistical and empirical investigation.

In the first section, I concentrate on utility-based definitions of substitutes and complements, as they can be reconstructed from the writings of the years 1880 to 1910. Those definitions will be linked with the main objectives of the writers (Auspitz and Lieben, Edgeworth, Marshall) (**1. The utility based construction of complementarity**)

In the second section, I focus on the preference-based approach and I show to what extent it was competing with the utility-based approach. The main writers here will be of course Fisher, Pareto and Johnson. (**2. The preference-based approach to complementarity**).

The aim of the two following sections is to insert reflections on complementarity into their theoretical environment. It will appear clearly that the utility-based definition was developed and used mainly within the context of consumer's surplus theory, whereas the preference-based

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with a new an intuitive cardinalist definition based on von Neumann's utility function. He discusses also some philosophical aspects of complementarity.

[3] It must be mentioned also that some writers will tackle substitutability solely through the idea of related market demands, without investigating the utility foundations for it. This way of seeing things is of minor importance for our story, but it will come to the forefront later, through the development of general equilibrium. It is thus to be found almost exclusively in Walras *Elements of pure political economy* (1874), without any serious relation with utility or preferences, but rather exclusively with needs. See also (Dalton, 1925)

[4] It is not easy to insert Slutsky's contribution into a history of economic thought, because it is well known that he was not followed until its rediscovery, and also because Slutsky himself does not provide a precise account of what he had been acquainted with at the time of the writing of the 1915 article (see Chipman and Lenfant, 2002)

definition was used to develop a theory of demand. This is the subject of section 3 (**3. Consumer surplus and complementarity**) and of section 4 (**4. Demand and complementarity**)

Section 5 deals exclusively with Slutsky's contribution, in order to delineate the analytical and methodological breakthrough he introduced in demand and utility theory, with peculiar attention to the consequences on complementarity (**5. Slutsky and complementarity: the breaking point**).

1. The utility-based construction of complementarity

The first analytical definition of complements and substitutes in consumption goes back to Auspitz and Lieben's *Untersuchungen über die Theorie des Preises* (1889) (hereafter *Investigations*). They were the first one to provide a stable and analytically useful definition of complementarity. Although Edgeworth (1881) seems to have been the first to deal with the cross second-order derivative of a utility function, he did not aim to speak about complementarity. Later, Marshall, Fisher and Pareto will have recourse to the Auspitz and Lieben criterion. I will take things, as far as possible, in chronological order.

1.1. Why Edgeworth did not invent complementarity

In *Mathematical Psychics* (1881), Edgeworth introduces the first generalized utility function, with only two variables, and he discusses on its basis the shape of the indifference curve. He is thus led naturally to deal with the second derivatives of the utility function and among them with the cross derivative. A modern reader would thus be tempted to attribute to Edgeworth the first analytical definition of a subjective relation between goods. But the fact is that Edgeworth stops short of interpreting the cross derivatives as the mathematical representation of complements and substitutes.

In the following, I show that Edgeworth had no need for a definition of substitutes and complements, because he did not want to make a theory of choice. Although he had to cope with the sign of the cross derivative of a utility function, complementarity is of no use for his utilitarian project.

In the first part of the book, which is devoted to the *Economical calculus*, Edgeworth presents a situation of bargaining between two persons. This situation of bilateral monopoly, without competition, is "indeterminate" (Edgeworth, 1881, 20). Through the construction of indifference curves, Edgeworth will show that the final result of the bargaining can be represented by an infinite set of "settlements": namely, the *contract curve*.

In his presentation of the model, Edgeworth introduces a non additive utility function with two arguments. Let $u(x, y)$ and $\varphi(x, y)$ represent respectively the total utility functions for Robinson and Friday.[5] Taking the conditions of agreement between contractors as given, with usual notation

[5] The functional form is more general than that of Jevons (1871). According to Edgeworth, indeed, "utility is regarded as a function of the two variables [*x* and *y*], not the sum of two functions of each" (Edgeworth, 1881, fifth appendix, 104).

for derivatives, Edgeworth obtains an equation whose solutions (x,y) describe the *contract curve*:

(1) $$u'_x(x,y).\varphi'_y(x,y) - u'_y(x,y).\varphi'_x(x,y) = 0$$

The aim of Edgeworth is then to show that the bargaining between two individuals can end at any point on the contract curve [6]. This demonstration will be based on the concept of indifference curve. Considering that the agent is indifferent to any exchange of dx against dy if the marginal utility of the quantity received equals the marginal disutility of giving the other in exchange, then one obtains the following equation for what Edgeworth calls an *indifference line*:

(2) $$u'_x dx + u'_y dy = 0$$

Given the indeterminacy of the contract in the case of bilateral monopoly, Edgeworth aims at showing that the rise in the number of agents will restrain the set of possible "final settlements". His demonstration rests upon the property of convexity of the indifference curve.[7] On this occasion, Edgeworth will refer to the sign of the cross second-order derivatives of $u(\cdot)$. The convexity of indifference curves is presented as a consequence of the fundamental axioms of the "Utilitarian Calculus": "the *law of diminishing utility*" and the "*law of increasing fatigue*" (Edgeworth, 1881, 6). Thus, when Edgeworth comes to the example of Robinson and Friday – a situation of exchange between work and money wages[8] – he gives the main properties of utility:

> "Here *x* represents the *sacrifice objectively measured* of X, it may be manual work done, or commodity manufactured, or capital abstained from during a certain time. And *y* is the objectively measured remuneration of X. Hence it may be assumed, according to the two first axioms of the Utilitarian Calculus, the law of increasing labour, and the law of decreasing utility, that [u] being the utility of X, [1] $\frac{du}{dx}$ is continually *negative*, $\frac{du}{dy}$ *positive*; [2] $\frac{d^2u}{dx^2}, \frac{d^2u}{dy^2}, \frac{d^2u}{dxdy}$, continually

[6] Each point on this curve corresponds to a rate of exchange dx/dy of which only one corresponds to Jevons' equation of exchange: $\frac{u'_x(a-x)}{u'_y(y)} = \frac{y}{x} = \frac{\varphi'_x(x)}{\varphi'_y(b-y)}$ (cf. Jevons, 1871, 100 ; Edgeworth, 1881, 109; and for a wider discussion Chaigneau, 1997, 181).

[7] The properties of the indifference curves allow Edgeworth "to develop a coherent presentation of the bargains that will take place when the market is composed of more than two agents" (Chaigneau (1997), 321). See also Chipman (1965, 42) and Creedy (1979, 166-9).

[8] It must be noted that the "goods" exchanged in Edgeworth's example are *work* and *wages*. They refer certainly to the generic content of the utilitarian postulates associated to those goods. Money is the good that allows obtaining means of pleasure, and work is the good that generates fatigue, pain and disutility. Meanwhile, the generic nature of the goods exchanged does not reduce the generality of the model, which is supposed to be appropriate also to any kind of exchange (see Chaigneau, 1997, 318 and 329-30). The postulates are representative of "a general and abstract point of view" (Edgeworth, 1881, 35).

negative. (Attention is solicited to the interpretation of the third condition)." (Edgeworth, 1881, 35).

From this, Edgeworth (1881, 35) will infer the convexity of indifference lines. From equation (2), the slope of the indifference line (the marginal rate of substitution) is given by (3):

(3) $$\frac{-dy}{dx} = \frac{u'_x}{u'_y}$$

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From (3) Edgeworth expresses the derivative of the marginal rate of substitution regarding the variable *x*, representing the rate of increase of the slope of the indifference line.

(4) $$\frac{d^2y}{dx^2} = \frac{-\left[u''_{x^2} + u''_{xy}\frac{dy}{dx}\right]u'_y + u'_x\left[u''_{yx} + u''_{y^2}\frac{dy}{dx}\right]}{\left[u'_x\right]^2}$$

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From the axioms of the utilitarian calculus, (4) is "perfectly positive" (Edgeworth, 1881, 36): "Therefore the indifference-curve (so far as we are concerned with it) is convex to the abscissa" (Edgeworth, 1881, 36). For all that, Edgeworth can be reproached for not making precise the relevant hypothesis for convexity. Indeed, Allen (1934, 119) showed that Edgeworth's hypothesis on the second-order derivatives are neither necessary nor sufficient for convexity, convexity being guaranteed by $u'_i > 0$ for all *i*. Hence

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Much could be said about Edgeworth's presentation of the laws of the utilitarian calculus. We will be content to mention that Edgeworth draws attention to the sign of u''_{xy} and that he will never come back to it later in the book. No specific discussion for this axiom is to be found, except in the fifth appendix which is devoted to a commentary of Jevons's equation of exchange. There, Edgeworth presents the new axiom of the utilitarian calculus as a variety of the law of diminishing marginal utility:

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"The rate of increase of utility derived from one sort of wealth diminishes with the increase of other sorts of wealth." (Edgeworth, 1881, 108)

To conclude, Edgeworth does not aim at investigating the relations between utility of one good and utility of another good. Such an interpretation would change indifference curves into a tool for choice analysis. On the contrary, the law of marginal decreasing utility is above all a general principle of the *utilitarian calculus*.[9] The reader of *Mathematical Psychics* must keep in mind that Edgeworth's aim is to justify the need for a principle of arbitration, and that he will be a proponent of a utilitarian principle. The two entities to be distributed among participants, according to the utilitarian principle, are means of pleasure and effort:

[9] For this reason, Stigler's (1950 (1965), 99) and Newman's (1987a, 95) interpretation of Edgeworth assumption as a hypothesis of competitive goods and Samuelson's (1974, 1279) interpretation of it as a complementarity assumption are equally misleading.

> "The rate of increase of pleasure decreases as its means increase. The postulate asserts that the second differential of pleasure *with regard to means* is continually negative." (Edgeworth, 1881, 61, italic ours)

According to Edgeworth, this postulate is self-evident, widely shared by theoreticians, and its empirical validity has been established by psychophysicians.[10]

In a few words, there is no room for complementarity in Edgeworth's project, to the extent that utility is not interpreted as enhancing a subjective relation between the goods of the contract, but a more generic relation between means of pleasure and pain. The interpretation of the sign of the cross second-order derivative of the utility function is part of a quite different project, that of promoting the need for a (utilitarian) principle of arbitration. *A contrario*, we see here that the introduction of a generalized utility function is not enough to establish a typology of substitutes and complements.

1.2. The Auspitz and Lieben criterion and its posterity

Tribute must be paid to Rudolf Auspitz and Richard Lieben, two Viennese mathematical economists, for providing the first mathematical criterion of substitutability on the basis of the sign of the cross second-order derivative of an introspectively felt utility function. This definition appeared in their main work, *Untersuchungen über die Theorie des Preises* (1889). Quite independently of Edgeworth (1881), Auspitz and Lieben represent utility as a generalized function, as opposed to the traditional additive function. The following is a very simplified presentation of Auspitz and Lieben's argument.[11] When necessary, for need of homogeneity, the notations have been changed.

In the first part of the *Investigations*, Auspitz and Lieben provide a provisional analysis of utility and demand, without accounting for substitutes and complements. Notably, they introduce what is called a "utility curve" (*Nützlichkeitskurve*). Good x_1 will be bought only to the extent that it brings some utility to the consumer:

> "This utility is measured by the maximum effort or by the maximum sum of money that consumers could devote without disadvantage to this quantity of [x_1]" (Auspitz and Lieben, 1889 (1914), 5)

[10] Edgeworth refers to Fechner, Delboeuf and Bain (for details, cf. Chaigneau, 1997, ch.4 and ch.5). When he grounds the utilitarian calculus on psychophysics, Edgeworth asserts once more the generality of the result: "All the formulae suggested for the relation between quantity of stimulus and intensity of sensation agree in possessing the property under consideration" (Edgeworth, 1881, 62). And later, Edgeworth reasserts the law of decreasing marginal utility: "In so far as a *part only* of happiness increases *only proportionately* to its means, the second differential of happiness with regard to means does not cease to be *continually* negative" (Edgeworth, 1881, 63).

[11] The reader interested in Auspitz and Lieben's work is reported to Schumpeter (1930), Jäggi (1992), Niehans and Jäggi (1993), Niehans and Jäggi (1995), Niehans (1987a and 1987b).

In the framework of an exchange of good x_1 against money, the "utility curve" in the plan (x_1, money-utility) is represented by a concave utility curve.

Interpreting the "utility curve" is not an easy task. Indeed, the authors put only the quantity x_1 on the abscissa, and so utility appears to be a function of x_1 only. In this case, total utility must be represented by an additive utility function, and the utility associated to x_1 represents only that part of total utility attributed to it. This framework is clearly at odds with the idea that Auspitz and Lieben adopted the generalized utility function. Another way to interpret the utility curve is that the quantity x_1 is associated to a bundle of other goods (x_2, …x_n), in fixed proportions, so that x_1 represents a composite commodity. This interpretation should not be retained. Indeed, Auspitz and Lieben make it precise that for every x_1 the consumer is able to adapt his bundle of other goods so as to maximize his total satisfaction. The third interpretation of the utility curve is that it represents the total utility derived from a bundle (x_1, x_2, …x_n) when (x_2, …, x_n) is perfectly adapted to the quantity x_1. This is actually Auspitz and Lieben's idea.[12]

From this utility curve, Auspitz and Lieben derive a "*demand curve*" in the (quantity / money) plane (Auspitz and Lieben, 1889 (1914), 281-282). Let x_1 be the quantity of the good, let $f(x_1)$ the function representing the *curve of utility*. Then, the functional relationship between x and the quantity of money given for it is $g(x_1) = x_1 f'(x_1)$. This comes directly from the fact that, taking the convention of a unitary marginal utility of money, the unitary price of good x_1 is given by $g(x_1)/x_1$ and must be equal to the marginal utility derived from its consumption, $f'(x_1)$. The shape of the demand curve *On*' is determined by the shape of the curve *On*. The curvature of the *constant utility curve* induces quite naturally a negative relation between price and quantity (Auspitz and Lieben, 1889 (1914), 13).

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[12] Two difficulties stem from this. Firstly, if the bundle (x_2, ..x_n) is varying with x_1, the consequence is that the total expenditure for this bundle is also varying. Accordingly, Auspitz and Lieben will 1) consider that the prices of all other goods $(p_i)_{i\geq 2}$ are given, adopting therefore a partial equilibrium framework; 2) they introduce the final cash balance R of the consumer as an argument of the utility function, 3) they suppose that the marginal utility of money is constant. Thus, the utility function can be written as a quasi-linear function as regards the terminal cash balance or wealth R: $u = \varphi(x_1, x_2, ..., x_n) + R$

Secondly, the utility curve can be interpreted also as an indifference curve. The utility curve is presented as the sum of money that might be given against x_1 "without disadvantage". The meaning of this is clearly that the utility curves is also an indifference curve within the framework of the exchange of money against good x_1, and with the special assumption of constant marginal utility of money (see Edgeworth (1891, 238)).

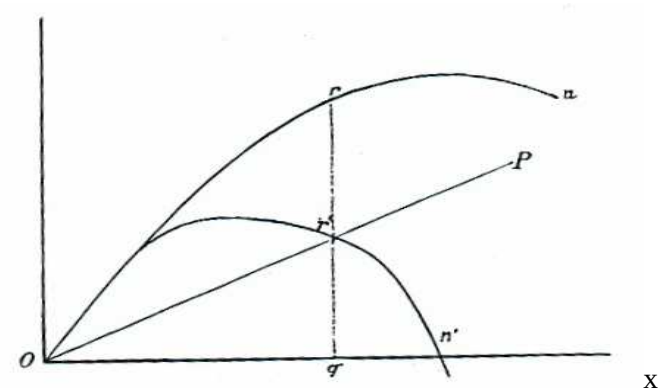

Fig. 1: Utility, demand, and consumer rent.
Auspitz and Lieben (1889, 391)

This is just a provisional analysis of the *utility curve*, based on the sole hypothesis of decreasing marginal utility. Auspitz and Lieben are fully aware of the necessity to cope with the idea that utility is a function of the consumption bundle in its most general form, and thus to provide "a more comprehensive study" of the *utility curve* (Auspitz and Lieben, 1889 (1914), 87).[13] Notably, they emphasize that a more precise study of *On* will prove rigorously this concavity (Auspitz and Lieben, 1889 (1914), 6).

In the second part of the book (*the constitutive elements of the curves*) Auspitz and Lieben provide a detailed study of the shape of the constant utility curve, accounting for interdependencies between utility. The mathematical exposition is given in the second appendix, especially in section 2.[14]

Let us consider utility as a quasi linear function with respect to income. Utility is given by:

(5) $$u = \varphi(x_1,...,x_n) + R = \varphi(x_1,...x_n) + R_0 - \sum_{i=1}^{n} p_i x_i \ ,$$

where R denotes the terminal cash balance given by the difference between the initial cash balance R_0 and the total expenditure for $(x_1,...,x_n)$ at prices $(p_1,...,p_n)$: $R = R_0 - \sum_i p_i x_i$.

Consider now R_1 the terminal cash balance when x_1 is free of charge. Then, $u = \varphi(x_1,...,x_n) + R_1 - p_1 x_1$. In this function, x_1 , R_0 and (p_1, …, p_n) are parameters, and the maximization of u relatively to n variables yields: $\partial\varphi(.)/\partial x_i = p_i$ for all i. Auspitz and Lieben's idea is then that the optimal values of $(x_i)_{i\geq 2}$ can be considered as known from the last equation, independently of the value of p_1. So, $\partial\varphi(.)/\partial x_i$ depends on x_1, and $(x_i)_{i\geq 2}$

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[13] The first chapter of the book was previously published in 1887 under the title: *Zur Theorie des Preises* (Leipzig: Duncker & Humblot)

[14] See Schmidt (2004) for a detailed analysis of Auspitz and Lieben's analysis of the utility function, emphasizing the use of the envelope theorem.

can be expressed as a function of x_1. Then, Auspitz and Lieben introduce the pleasure function (*Lebensgenusskurve*) $f(x_1)$ defined as follows:

(6) $$f(x_1) = \varphi(x_1, x_2(x_1), \ldots x_n(x_1)) + R_1$$

By derivation of $f(x_1)$, using the chain rule:

(7) $$f'_{x_1}(x_1) = \frac{\partial \varphi(.)}{\partial x_1} + \sum_{i=2}^{n} \left(\frac{\partial \varphi(.)}{\partial x_i} - p_i \right) \frac{\partial x_i}{\partial x_1}$$

Using $\partial \varphi / \partial x_i = p_i$ yields $f'(x_1) = \partial \varphi / \partial x_1$. Auspitz and Lieben go on to discuss the shape of the curve of pleasure:

(8) $$f''(x_1) = \frac{d^2 \varphi(\cdot)}{dx_1^2} + \sum_{i=2}^{n} \frac{\partial^2 \varphi}{\partial x_i \partial x_1} \frac{\partial x_i}{\partial x_1}$$

The first term of this expression is negative. From the fact that the consumer is free to adapt his bundle to his advantage, the sum of terms into square brackets must be positive, but it may be that some of them are negative. On this occasion, Auspitz and Lieben introduce their famous criterion of complementarity. Considering the cross second-order derivatives $(\partial^2 f(\cdot) / \partial x_1 \partial x_i)_{i \geq 2}$, they will be positive, zero, or negative

> "depending on whether (x_i) is a complement to the enjoyment of (x_1), is totally indifferent to (x_1), or is competing to it."(Auspitz and Lieben, 1889 (1914), 319).

Then, Auspitz and Lieben make use of the general assumption that the rise in x_1 will be accompanied by a rise in the quantity of its complementary goods, and with a decrease in the quantity of its competing goods.[15] Otherwise stated, the cross derivatives multiplied by the corresponding variation of demand, $\frac{\partial^2 f(\cdot)}{\partial x_1 \partial x_i} \cdot \frac{dx_i}{dx_1}$ will be positive (Auspitz and Lieben, 1889 (1914), 319). Consequently, the *utility curve* is less concave when each individual is free to adapt his consumption in the most favorable way.[16]

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As a partial conclusion, one can say that Auspitz and Lieben introduce the utility-based criterion of complementarity on the occasion of the analysis of the utility curve, and they never use it again formally elsewhere in the *Investigations*. On the contrary, they will use the concept of

[15] Chipman (1977) has shown that if all distinct commodities are Auspitz-Lieben complements of each other, then the law of demand must hold: income elasticities of demand must be positive and demand for a commodity must be a decreasing function of its own price. For other empirical implications of the Auspitz-Lieben criterion, see Weber (2000).

[16] The authors have to cope with another difficulty: that the sign of the whole term $f''(x_1)$ may become positive, thus allowing for convex parts of the constant utility curve. Auspitz and Lieben do not hesitate to exclude convex parts of the curve on the grounds that it is incompatible with the condition of equilibrium. Thus, the possibility of a convex curve is ruled out. Anyway, Auspitz and Lieben (1889 (1914), 317) take it for granted that the "physiological law" of diminishing marginal utility "exerts a preponderant influence on every curve." (Auspitz and Lieben, 1889 (1914), 316). Cf. also Auspitz and Lieben (1889 (1914), 7).

complementarity, in a broader sense, in relation with consumer's surplus theory (see section 3).

1.3. Marshall and the utility-based notion of complementarity

Marshall's first preoccupation with complementarity can be traced back to *The Economics of Industry*. In his early writings (*The Essay on Value* (1870-71) and *The pure Theory of domestic values* (1879)), Marshall's analysis on utility deals only with the definition of the *value in use* of a thing, and with the measurement of this value in use by money. Value in use is taken as an equivalent for a quantity of pleasure (Whitaker, 1975, I, 125), and it is measurable through the monetary sacrifice that someone is liable to make for it (Whitaker, 1975, I, 128). *The Economics of Industry* testifies for a kind of break with the Marshallian conception of utility. Of course, he still regards "*value in use* or utility of a thing to a person as the amount of pleasure or satisfaction which [this person] derives from possessing it" (Marshall and Marshall, 1879 (1884), book II, chap.1, §4, 68). At the same time, he displays much more attention to the improvement of utility theory. Noteworthy, he will take into account that goods may have similar uses,[17] and the idea that *final utility* depends on the availability of alternative consumption is quite explicit (Marshall, 1879 (1884), book II, chap.1, §6, 70)

Nevertheless, Marshall will always privilege the representation of total utility as an additive separable function.[18] It is well known that Marshall takes an additive utility function as a good representation of total utility. In the note XII bis of the Appendices of the *Principles*, he reckons to the review he made of *Mathematical Psychics*, and this note will stay unchanged until the last edition of the *Principles*:

> "Prof. Edgeworth's plan of representing U and V as general functions of *x* and *y* has great attractions to the mathematician; but it seems less adapted to express the everyday facts of economic life than that of regarding, as Jevons did, the marginal utilities of apples as functions of *x* [the quantity of apples] simply." (Marshall, 1898, 798)

As an application of this principle, Marshall argues that regarding the measurement of consumer's surplus, goods that have the same use can often be grouped together and treated as one single composite good:[19]

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[17] "The Utility of anything to a man, its power of satisfying his wants, depends partly upon the quantity of *things of the same kind* that he has already. The more he has of it the less will be the utility of more of it to him." (Marshall & Marshall, 1879 (1884), book II, chap.1, §5, 69, italics ours)

[18] From an ordinalist standpoint, it means that there must exist at least one monotone transformation $F(\cdot)$ so that $F(U(x_1, x_2, ...x_n) = u_1(x_1) + u_2(x_2) + ... + u_n(x_n)$

[19] The same principle applies also to complementary goods: "On the other hand, if we have calculated the total utility of fuel with reference to the fact that without it we could not obtain hot water to obtain the beverage tea from tea leaves, we should count something twice over if we added to that utility the total utility of tea leaves, reckoned on a similar plan." (Marshall, 1898, 206 note 1)

> "The loss that people would suffer from being deprived both of tea and coffee would be greater than the sum of their losses from being deprived of either alone: and therefore the total utility of tea and coffee is greater than the sum of the total utility of tea calculated on the supposition that people can have recourse to coffee, and that of coffee calculated on a like supposition as to tea. This difficulty can be theoretically evaded by grouping the two "rival" commodities together under a common demand schedule." (Marshall, 1898, 206, note 1)

Those few passages exhaust the theory of substitutes and complements as it was developed in Marshall's work. For all that, it is not easy to infer that Marshall adheres to the mathematical definition of substitutes and complements, although he would certainly have accepted it.[20]

Simply, he will take care to formulate a law of demand taking account of substitutes in consumption. Implicitly, there is the idea that marginal utility of a good is in fact the marginal utility of a composite good, evaluated against the quantity consumed of this good or indifferently against the quantity of any other substitutable good. This is an *ad hoc* astuteness for measuring consumer's surplus, not a definition of substitutes and complements. We shall see later how Marshall integrates substitutes and complements into his theory of consumer surplus.

1.4. Some other contributions to the utility-based approach, with special focus on Pareto.

Here, I would like to come back to Auspitz and Lieben's followers who investigated the utility-based approach, especially Pareto. Indeed, whereas Pareto is known for his use of indifference curves, Pareto's first and most profound analysis of complementarity originates, in all probability, in Auspitz and Lieben's concept. It might also have some links with other contributions to mathematical economics, especially with Pantaleoni's *Principii di Economia Pura* (1889).

Auspitz and Lieben's criterion anticipates by many years the identical criterion used by Pareto (1909) and Edgeworth (1915). In fact, it is not quite sure that Pareto and Edgeworth actually *adopted* the Auspitz and Lieben criterion. Actually, the definition was first explicitly taken from Auspitz and Lieben by Fisher (1892, 65). As for Edgeworth and Pareto, although both of them knew about Auspitz and Lieben's book, neither one of them credited the two Viennese bankers with this definition (see Edgeworth (1897 (1925), 117n), Edgeworth (1915 (1925), 464), Pareto (1909, ch.IV, 264-265 and Appendix, §46, 575 and §124, 654); see also Chipman and Lenfant (2002) and Lenfant (2005)).[21]

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[20] In fact, it comes to the same thing, because Marshall is just saying that in the context of perfectly complementary goods, the marginal utilities of the two goods cannot simultaneously be positive.

[21] Regarding Pareto, particularly, we know from his correspondence with Pantaleoni that he didn't read the German language but that he was provided by Pantaleoni a copy of the book as well as a (partial or complete?) translation of it. At the time, Pareto had been working on an article on the polemic between Walras and Auspitz and Lieben (Pareto, 1892), so it is doubtful that he would not have read it carefully.. See Pareto (1960, 147, 150-152, 157,

It is only in 1906, in the first (Italian) version of the *Manual of political Economy*, that Pareto undertakes a thorough analysis of complementarity, and more broadly, of utility interdependencies and tastes.[22] It is the object of the fourth chapter of the book, entitled "Tastes", at the end of which Pareto would adopt the Auspitz and Lieben definition. Nevertheless, through his analysis, Pareto will change in depth the meaning of complementarity for value theory. And this is my point: with Pareto, complementarity becomes a concept for the theory of choice, and beyond that for price theory.

Pareto (1909 (1971), 182, §8) engages in a discussion about complementarity through the introduction of a generalized utility function (Pareto, 1909, (1971), 183-4).[23] In the *Manual*, the interdependence between marginal utilities is not interpreted directly through the idea of complementarity. Pareto takes a broader typology. First, he distinguishes between two "species of dependence":

> "1° one which arises from the fact that the pleasure from one consumption is connected with the pleasure from other consumption; 2° one which manifests itself in the fact that one thing may be substituted for another to produce sensations, if not identical, at least approximately equal." (Pareto, 1909 (1971), 182-3).

This typology overlaps the Auspitz-Lieben-Fisher presentation, but it is more subtle. Indeed, Pareto will identify two aspects within the first species or type: the case [1a] when dependence results "from the fact that we appreciate the use and consumption of a thing more or less according to the circumstances in which we find ourselves" (Pareto, 1909 (1971), 183), and the case [1b] when dependence arises "from the fact that certain things must be used jointly in order to yield us pleasure" (Pareto, 1909 (1971), 183). Only in the case [1b] can one examine a relation between goods, relatively to a given person for given tastes and state of mind, and only in this case it is correct to speak about "COMPLEMENTARY GOODS" (Pareto, 1909 (1971), 183). The same remark is relevant to the second type of dependence "between goods which correspond to a certain need" (Pareto, 1909 (1971), 185). Pareto distinguishes between an equivalence "relative to the man's tastes, or to his needs" (Pareto, 1909 (1971), 185). I think that this distinction is of primary importance, because it is at the watershed between a purely subjective representation of preferences and a more objective construction, based on the identification of needs. Thus, in the case of a taste-based equivalence [2a] "If the relation of equivalence refers strictly to the tastes of the individual, it is nothing else but the relation which gives the indifference curve for equivalent goods; hence it is unnecessary to make a separate study of it" (Pareto, 1909 (1971), 185). In the case of a needs-based

196) and also Pareto's comments on the geometrical representation of the constant utility curves and of the demand curves from Auspitz and Lieben (1889) in Pareto (1960, 467-70).

[22] In his first contribution to pure economics, "Considerazioni sui Principii Fondamenti dell' Economia Politica Pura" (Pareto, 1892-1893), Pareto already tackled the question of complementarity, and again in the *Cours d'économie politique* (1896-1897). In both references, Pareto's treatment is very simple and has a Marshallian flavor, in the sense that it reduces substitutes to perfect substitutes and gathers together complements.

[23] Pareto (1892-1893, May 1892, 411) mentions Edgeworth's anteriority.

equivalence [2b] "there would no longer be an identity between the equivalence relationship and the indifference curve" (Pareto, 1909 (1971), 186).

This discussion is fundamental to the extent that it gives simultaneously a meaning to the terms "complements" and "substitutes" and a meaning to the concept of preferences and indifference curves. Pareto's conception of preferences as a picture of tastes disencumbered of the "pressure of necessity" is quite original compared with any other discussion of the same subject at the time. Nevertheless, Pareto will assume similar properties. In the cases [1a] and [1b] of complementarity, "in general, for the first type of dependence, the elementary ophelimity of A increases when the quantities of certain goods B, C, … increase" (Pareto, 1909 (1971), 194). On the contrary, in cases [2a] and [2b], "if A can replace a good B, the elementary ophelimity of A will be smaller insofar as one has a larger amount of its substitute B" (Pareto, 1909 (1971), 194, §41). Mathematically, this comes exactly to the Auspitz-Lieben-Fisher criterion. With $\varphi(x_1, x_2, ..., x_n)$ the function of utility or ophelimity of a basket, complementary goods (dependence of the first type) gives $\varphi_{ij}(\cdot) > 0$ for all $i \neq j$ and substitutes (dependence of the second type) gives $\varphi_{ij}(\cdot) < 0$ for all $i \neq j$.[24]

Code de champ modifié

Code de champ modifié

From the preceding, one can say that Pareto comes to the "traditional" (Auspitz-Lieben) criterion through an original study of tastes, clarifying his own ideas about needs and tastes.

Much has been written on Pareto's evolution from a utilitarian-cardinalist point of view to an ordinalist stance. Pareto's treatment of complementarity embodies a shift toward a preference-based approach. Actually, Pareto's presentation of complementarity is developed along broader lines in the Appendix. Whereas the main text of the *Manual* pedagogically insists on the properties of ophelimity and then illustrates the usefulness of the indifference curve for choice theory, the Appendix goes in the reverse order.[25] Anyway, Pareto recognizes that he does not have any credo about the best way to proceed: "Economists began by appealing to experience for the characteristics of ophelimity; they then deduced indifference lines from those characteristics. The reverse route *may* be followed" (Pareto, 1909 (1971), 195).

From this, we can assume reasonably that Pareto is more representative of a preference-based approach to complementarity.[26]

[24] Pareto (1909 (1971), Appendix, §46, equations 63 and 64 respectively). In the French edition, the inequalities (63) and (64) have been inverted. This fault has been corrected in the American translation.

[25] First, the question is about the main properties of indifference lines (§§44-45) and then comes the question of the characteristics of the indices of ophelimity related to it (§§46-51). Already in the main text, the discussion of preferences and that of utility indices are intermingled.

[26] A few words are in order about Pantaleoni's *Principii* (1889) as another source of reflection on complementarity. Pantaleoni's book was important in Italy, introducing marginalism and mathematical economists to Italian economists. It is clear from the reading of the book that Pantaleoni was well informed of the developments of marginalism throughout Europe. The first part of the book is entirely devoted to the "Theory of utility".

2. The preference-based approach

Fisher (1892) and Pareto (1909) were at the origin of a geometrical conception of complementarity based on the concept of indifference curves. The preference-based approach is not presented as breaking the utility-based approach, but rather as an improvement of it. Actually, there are some important differences that I shall point out.

In this section, I aim at showing that the development of the preference-based approach is associated with the search for possible links between the properties of preferences on the one hand – whatever the justification given for them – and demand behavior. Of course, that there is a relation between preferences and demand is quite evident. My point is that complementarity was at the heart of the investigations on this issue. This aim was only secondary in the utility-based approach, and it came really to the forefront with the use of indifference curves in microeconomics. The generalization of the use of indifference curves was at the starting point of a profound methodological transformation of mathematical economics, towards a more experimentally founded theory of choice. Fisher and Pareto, through the introduction of an ordinalist representation of utility, will lead to a change in the theoretical use of complementarity.

2.1. The geometrical typology of Irving Fisher

Fisher's (1892) main departure from Auspitz and Lieben (1889) consists in adopting a general equilibrium point of view. But, in the same spirit as the two Viennese, Fisher will use their definition in order to associate subjective relations between goods to objective relations in terms of market behavior. Whereas Auspitz and Lieben opened the way toward a sophisticated analysis of utility and choice, within a partial equilibrium framework, Fisher will provide a sophisticated representation of a walrasian general equilibrium of exchange, through the introduction of a generalized utility function into it.

We find here many things that will serve as a basis for Pareto's analysis of tastes. First, following Spencer's social darwinism of *The data of ethics*, Pantaleoni (1889 (1931), 25) insists on the psychical and psychological circumstances affecting utility. Secondly, when dealing with marginal utility analysis, Pantaleoni draws attention to Auspitz and Lieben (1889) (as well as to Giovani Batista Antonelli's 1886 memoir) (Pantaleoni, 1889 (1931), 105-108). Thirdly, Pantaleoni (1889 (1931), 110-111 and 127) introduces to the notions of "complementary goods" and "equivalent goods". The former refers to the fact that things will have no use, relatively to a given need, if they are not joint in given proportions with others. Those things, once associated towards a given use, provide a "complementary utility", whereas their utility would have been otherwise negative. As for "equivalent goods", he suggests to construct tables of equivalence for wide categories of goods, and idea anticipating on Pareto's remark about "equivalence in terms of needs". So, Pareto must have had in mind Pantaleoni's foundations together with Auspitz and Lieben's mathematical criterion for investigating utility interdependencies. His main originality, therefore, is rather in the preference-based approach.

On many occasions, Auspitz and Lieben's analytical influence would appear evident.[27] Fisher is the only one, to our knowledge, that gave full credit to the Viennese bankers for their definition of substitutes and complements in consumption (Fisher, 1892, 65). In the first part of the *Mathematical Investigations*, Fisher gives the system of equations of exchange of a general equilibrium, in the spirit of Walras.[28] The originality of Fisher consists essentially in catching the analytical consequences of the introduction of a generalized utility function:

> "It is needful here to distinguish carefully between two ways in which the quantity of one commodity can affect the utility of others. Even under the supposition of part I [independent utilities], a change in the price of clothes effected a change in the individual valuation of money and so changed the quantity of bread consumed and so in turn changed the marginal utility and price of bread. But under our new supposition [generalized utility], a change in the price of butter directly changes the utility of the same quantity of bread. In the first case marginal utility of bread can change only after a change in its *quantity*. In the second the marginal utility of the *same* amount of bread changes." (Fisher, 1892, 64).

Later, Fisher uses Auspitz and Lieben's definition to discuss the shape of utility hills in the case of two goods (Fisher, 1892, 68-69). But in a first step, Fisher illustrates the theoretical interest of this typology through an example with three goods : bread, biscuits, and butter. He supposes that bread and biscuits are substitutes, whereas bread and butter are complements, and he wonders about the effect of a rise in the quantity of bread available in the economy. His argument is on the basis of an hydraulic machine, made of many cisterns representing the marginal utility levels :

> "If the quantity of bread is increased, the cisterns for biscuit may shrink and those for butter widen. That is the ordinate (marginal utility) for the *same quantity* of biscuit decreases, and of butter increases. The general effect is to keep the ratio of marginal utilities of bread and biscuit and so also their prices nearly constant, while the cheapening of bread may directly increase the marginal utility and price of butter irrespective of its quantity." (Fisher, 1892, 65).

More, Fisher is not content with those polar cases, and he goes beyond them to build a more general typology of complementarity on the basis of a geometrical representation of indifference curves.

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[27] "The two books which have influences me most are Jevons: '*Theory of Political Economy.*', and Auspitz und [*sic*] Lieben: 'Untersuchungen ueber die Theorie des Preises.' " (Fisher, 1892, 3-4). Fisher will reiterate later (Fisher, 1915, 348-350).

[28] Fisher notes the similarity between Walras system and his own system: "These equations are essentially those of Walras in his *Éléments d'économie politique pure*. The only fundamental differences are that I use marginal utility throughout and treat it as a function of the quantities of commodity, whereas professor Walras makes the quantity of each commodity a function of the prices. That similar results should be obtained independently and by separate paths is certainly an argument to be weighed by those skeptical of the mathematical method" (Fisher, 1892, 4).

Here, we enter fully into a preference-based approach to complementarity. This approach is based on the idea that, within a two-goods framework, the complementarity relations between goods could be made apparent and classified through a study of the curvature of indifference curves. At least, this was the first analysis of this type. And it was to attract more and more attention as the ordinalist methodology was progressing.

Nevertheless, the idea of interpreting complementarity on the basis of the shape of an indifference curve may seem rather strange. If more or less convexity is to represent by degree more or less substitutability, then the representation of complementary goods will be expelled from the field, except in a polar case of perfect complementarity. After all, one can say that this is just a question of words. The point is rather that Fisher is not interested in complementarity in itself; indeed, his idea is to develop a continuity of representation from complements to substitutes and to implement it into price theory.

Given a utility function with two arguments, Fisher calls "indifference curve" the curve corresponding to a set of points that have the same index of total utility: "They in general form a family of concentric curves vanishing finally at the point M of maximum satisfaction".[29]

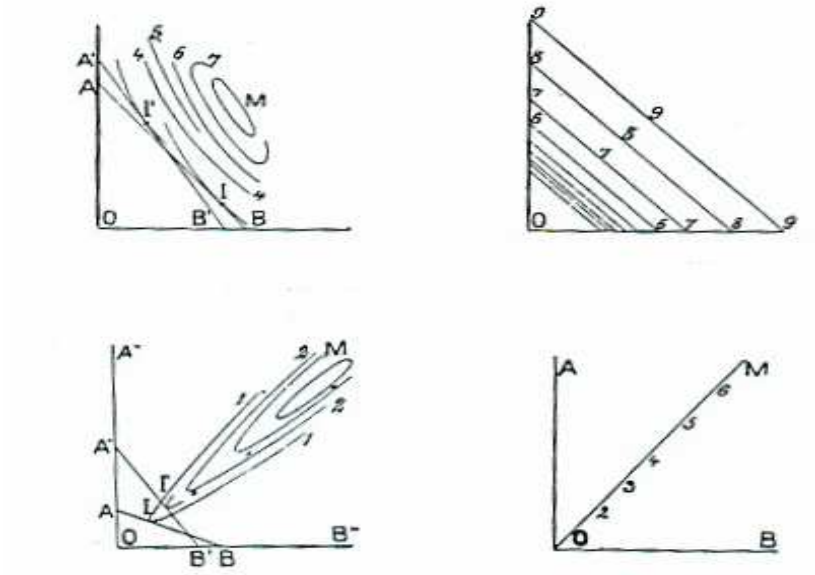

Figure 2 : Highly substitutable goods, perfect substitutes, Highly complementary goods, perfect complementary goods. Source : Fisher (1892, 71)

[29] It sounds curious that Fisher's analysis owes officially nothing at all to Edgeworth. The one and the other give the same name to the same analytical tool. Fisher mentions that he got acquainted with *Mathematical psychics* only "three days after part II was finished." (Fisher, 1892, 4). The only analytical connection he admits is about the use of a generalized utility function (Fisher, 1892, 4). This reluctance to recognize Edgeworth priority on many points can result from Fisher's opposition to the utilitarian spirit of Edgeworth construction (Chaigneau, 1996).

Fisher does not provide any justification for this geometrical typology, but it can be suspected that it is derived from the polar cases. It can be remarked also that the continuity between substitutes and complements is in fact independent of the case of perfect complements and substitutes, and that the perfect complements have nothing to do with the standard representation. Fisher does not rule out the part of the indifference curves that are not convex, even though he is aware that the choice will take place on a convex to the origin portion of the curve. Complements and substitutes are represented by circular indifference lines around a point of saturation (other things being constant). Those circular curves are all the more streched as the goods have complements or substitutes to each other to a high degree. In the case of complements, the convex to the origin part is the smallest.

That Fisher does not enter precisely into the explanation of its construction is not very astonishing. Fisher, among the early ordinalists, is the most suspicious about the introspective meaning of the concept of indifference curves. I would even say that he makes an instrumentalist use of the concept. For what is important, from the typology, is the continuity from one polar case to the other, but certainly also the implications of this typology for the price system. Indeed, in the final analysis, it is quite evident that Fisher's preference-based construction is not aimed at providing a subtle subjective typology of substitutes and complements. Obviously, this construction is rather crude and it is aimed at providing rather general and clear cut properties to the price system. This point will be developed later, in section 4 below, but it is clear from this that one of the methodological questions at stake is: on what kind of preference-based properties should a useful and significant typology of complementarity be erected?

Nothing allows Fisher to establish a connection between the utility-based criterion and the preference-based criterion, and he never provides any justification for this. A provisional conclusion on Auspitz and Lieben and on Fisher is in order. What is at stake, through the search for complementarity, is to build a bridge between subjective properties (in terms of utility or preferences) and an objective one, in terms of demand behavior or in terms of properties of the price system.

A third step is taken by Pareto, when erecting a far more sophisticated analysis of complementarity, appropriate to the development of a theory of choice.

2.2. Pareto and the preference-based approach

As has been said previously, Pareto paid attention to a description of preferences using indifference curves. In presenting complementarity Pareto seems to insist on the theoretical distinction between needs and tastes, but the examples given in the *Manual* are not very convincing (Pareto, 1909 (1971), 183, §10 and §12).

Theoretically, complementary goods are defined independently of any kind of psychical or physical needs. Whether glasses and bottles of wine are complementary may well be a pure question of preferences, but it may

depend also on the circumstances.[30] Pareto's theoretical distinction between equivalence in terms of tastes (preferences in the strict sense according to Pareto) and equivalence in terms of needs is not very easy to put into practice. What can be drawn from this is that Pareto identifies some "objective" constraints of choice that will often come to the forefront in his study of the individual and market demand behavior.[31] Pareto implicitly recognizes that it is a vain task to consider preferences independently of needs. Practically, he proposes (Pareto, 1909 (1971), 187, §21) to establish equivalencies, in terms of needs and tastes, of different baskets of bread, potatoes, kidney beans, meat, … relatively to an initial state of equilibrium. Otherwise stated, Pareto proposes to construct, experimentally, indifferences curves, that is preferences in a broader sense including tastes and needs.[32]

Another idea on the distinction between needs and tastes deals with the short run conception of indifference curves. In his example on coffee and tea, he draws indifference lines at right angles, showing that those goods are poorly substitutable, purely in terms of preferences. Preferences will always have this general shape, even though Italians should progressively drink much more tea and much less coffee.[33] It will not have any impact on the shape of the indifference line, but only on the position of the indifference map.

That Pareto investigates both utility theory and preferences is quite coherent with his methodological ecumenism, and with the specificity of the social sciences as compared with the natural sciences:

> "In political economy (…) our hypotheses deviate in part from reality; and consequently the results we can deduce from them will be able to correspond to the facts only within certain limits. (…) Theory of such sort is more often a mode of research than of proof, and we should never neglect to check whether the deductions correspond to reality" (Pareto, 1909 (1971), 190-191).

Furthermore, the preference for one way on the other may be a question of circumstances and convenience. Thus, Pareto remarks that in the most general case of interdependent utilities "the study of indifference lines gives us results which we would achieve less easily, for the moment at least, by recourse to experience alone in determining the characteristics of ophelimity" (Pareto, 1909 (1971), 196).

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[30] Pareto's general argument (1909 (1971), 184-185, §12) is that, within slight variations of the actual state of the consumption (drinking the same coffee in coffee cups of slightly different qualities), the ophelimity of coffee may well be independent of the coffee cup.

[31] See for instance Pareto (1909 (1971), 186, §§17-19)

[32] "We only study what occurs in a small region surrounding the concrete phenomenon which provides the factual data necessary to construct the theory" (Pareto, 1909 (1971), 190)

[33] The representation of the change in the indifference curves is not very coherent. Pareto would have better represented the dotted indifference lines in the neighborhood of the plain lines, and compared the consumption under the same relative prices (see Pareto, 1909 (1971), 189, fig 28).

Let us enter more fully in the technical properties of indifference curves. According to Pareto, indifference curves are decreasing and convex to the origin. In case of strictly completing goods, indifference curves are at right angles. In case of approximately complementary goods, the indifference curves have "a very slightly rounded angle" (Pareto, 1909 (1971), 202). The angle may well be "more rounded" for larger quantities of each good . And then, as the consumer approaches saturation, the angle tends to be a right angle (Pareto, 1909 (1971), 202, fig. 33). Coming to the second type of dependence (substitutes), Pareto develops the case of two substitutable goods, one of which is inferior to the other. The consumer can mix both goods up to a level of satiation, so that indifference curves become horizontal when the line of saturation is reached.[34] The last case studied is when goods are perfect substitutes (Pareto, 1909 (1971), 280, fig.36). In the case of the exchange of a good against money, Pareto remarks that the marginal rate of substitution is a decreasing function of the quantity of money, the quantity of the other good being given. Otherwise stated, the tangent to the indifference lines will be more inclined as the quantity of money is raised (Pareto, 1909 (1971), 206, fig.37). This property seems to be extended to any group of commodities taken as a substitute for money. This representation can be usefully compared to Fisher's case when there are two substitutes with an inferior and superior good. Pareto rightly recognizes that only rich people will be able to consume a positive quantity of a good when it is expensive and that only a decrease of the price will allow new consumers to buy that good (Pareto, 1909 (1971), 205-206, §65). So, this representation of the indifference curves is taken as a good approximation because it can lead to empirically reasonable conclusions: "And, as everyone knows, this is indeed the way things are" (Pareto, 1909 (1971), 206).

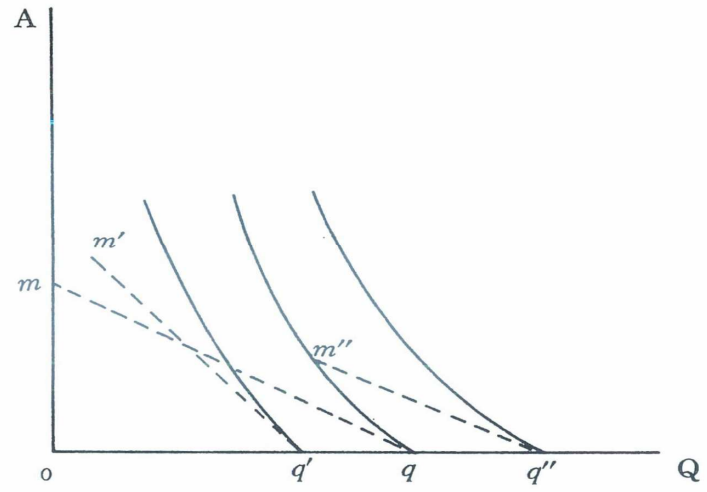

Fig.3: Wealth and consumption.

[34] Compare with Fisher (1892, 73, fig.24) and later Allais (1943). The graphic of fig. 33 in the American translation presents one flaw. The bottom indifference curve should become horizontal only after its crossing with the saturation straight line.

(Source: Pareto, 1909 (1971), fig.37 p. 206))

Finally, Pareto devotes much more time to investigating what we would call "peculiar cases" or exceptions. The standard case of convex indifference curves asymptotic to the axis (or even parallel to the axis), is just presented quickly as a *non-standard* case, because it implies that the consumer will consume both goods, even if one of them is very expensive, irrespective of his wealth. This does not "correspond to the majority of common goods" (Pareto, 1909 (1971), 207).

What emerges from Pareto's treatment of complementarity according to the preference-based approach is that he goes beyond a mere discussion of the shape of indifference curve, ant that he endeavors to describe the properties of the indifference map. This analysis goes with more depth into subtle analysis, in contrast with Fisher's case which did not intend to enter into such considerations.[35]

Thus, we see that Pareto's discussion of the indifference map does not turn around the opposition between substitutes and complements *per se*, but that it turns around another and more subtle distinction between asymmetric substitutes (that is substitutes of different qualities, one good being inferior and the other superior). A rationale for this is that through this distinction between inferior and superior goods, Pareto sought to present the occurrence of increasing demand.[36] So, it does not seem necessary to take the case of asymmetric substitutes as an example, except maybe as a pedagogic example.

For the first time, with Pareto, complementarity has to do with the shape of indifference maps and not with the shape of a single indifference curve. Consequently, Pareto seems to accept to focus more on the equations [57] than [56] to discuss complementarity: "Hence some further observations are necessary to clarify this matter. They probably lead to establish several categories of goods having a dependence of the second kind" (Pareto, 1909

[35] The properties of the indifference curves are also investigated in the Appendix §44 (Pareto, 1909 (1971), 416-8). In the (x,y) plane, the indifference curve $y = f(x)$ is decreasing, convex and, "save for some exceptional cases" (416) $d^3y/dx^3 < 0$ (Pareto, 1909 (1971), 416, inequality [56]), which means that the marginal rate of substitution decreases at an increasing rate. This, at least, is not always the case for substitutes. To discuss this, Pareto considers the shape of indifference maps. In general, the marginal rate of substitution (taken as a negative value) will increase along the *ox* axis, and the marginal rate of substitution will decrease (in negative values) along the *Oy* axis ($\delta_x \frac{dy}{dx} > 0$ and $\delta_y \frac{dy}{dx} < 0$, inequalities [57]). Nevertheless, Pareto will discuss in greater details those properties and link them with the above property that $d^3y/dx^3 < 0$. The discussion is a bit confusing. On the one hand there is no reason why those properties should be linked with the Auspitz and Lieben criterion. On the other hand, for a given utility function, there is no reason that d^3y/dx^3 should be everywhere negative. More, according to Pareto, the general property on the indifference map is only exhibited by independent goods and complementary goods.

[36] Pareto refers to this case as a case for substitutes, what is right, but he insists on the situation of asymmetric substitutes, whereas this is the case also for very symmetric substitutes.

(1971), 418). This is anyway a great step forward compared with Fisher's focus on indifference lines alone.

Once more, Pareto's methodology is at stake. It is as much important to go from demand behavior to preferences relations that generate them than to go from utility relations and preferences relations to the demand behaviors they imply. Each of the theoretical steps is as important as the other:

> "It is not so much the direct as the indirect observations which may be useful. Following the example of what is practiced in the physical sciences, we must make different hypotheses about the magnitudes in (57) and compare the inferences from these hypotheses with reality" (Pareto, 1909 (1971), 418) [37]

This typology is clearly driven and embedded in the theory of demand that it is aimed at developing and making more precise. Pareto is really the first to extract the concept from its first opposition between substitutes and complements. More, one can even say that the search for a new definition of substitutes and complements did not originate in the necessity to give up the cardinalist criterion and to develop an ordinalist one, but rather in the desire to find an operational concept relatively to demand theory.

2.3. William Ernest Johnson

Johnson's 1913 article ("The pure theory of utility curves") is often praised for being among the great articles that contributed to the development of a modern theory of consumer choice. Indeed, Johnson investigates in depth the relationships between utility, preferences and demand, in a way that fits with Pareto's contribution. William Ernest Johnson (1858-1931), logician and philosopher, is a pure product of Cambridge, where he studied and made his career. Although he taught mathematical economics, he wrote only three articles in the field of economics (Zabell, 1987). The 1913 article was remarkable enough to catch Edgeworth's attention (Edgeworth, 1915) and to figure in good place among the important contributions to mathematical economics.

It is now well known that Johnson gave the first ordinal definition of complementarity (Schultz, 1938; Stigler, 1950 (1965); Samuelson, 1974, Newman, 1987) based entirely on the shape of indifference maps, and it is clearly different from the traditional criterion.

Johnson's originality should not be exaggerated for all that. Compared with Pareto, Johnson is just making a broader typology of indifference maps after the manner of Pareto. To this aim he uses in all cases the marginal rate of substitution (but not the name of it).[38] He aims also at going beyond the polar cases of perfect substitutes and perfect complements. All the

[37] There are still some questions about Pareto's treatment of complementarity. Why should indifference curves have the same properties in the case of independent and in the case of complementary goods. Pareto does not give evidence for this contention. Anyway, it is clear that he must have in mind a utility function exhibiting the Auspitz-Lieben criterion.

[38] "What is needed is a representation of the *ratio* of one marginal utility to another. In fact, this ratio is precisely represented by the *slope* at any point of the [constant] utility curve" (Johnson, 1913, 103)

definitions in the article are based on the marginal rates of substitution of *x* to *y* or of *y* to *x*.

The fundamental breakthrough of Johnson (1913) is that, more than Pareto, he will link demand behavior to the shape of indifference curves, and especially, to the variations from one curve to the other. Whereas Pareto mixes arguments related both to the utility-based and to the preference-based definitions, Johnson systematic recasting of the concept leads to the subsumption of the utility-based criterion under the preference-based criterion. Accordingly, Johnson's main accomplishment is that he catches the analytical consequences of this. It is meaningful to study the effect of a price change and also to study the effect of an income change (prices being constant), and to discuss, in each cases, the shape of the curve and the quantity hold of each good. The difference with Pareto is only slight. It deals with the fact that Pareto analyses the expenditure effect (the effect of an exogenous income change) in a context of asymmetric substitutes (hierarchy of goods) whereas Jonhson does not privilege this kind of representation. In this respect, his typology of complements and substitutes is even more dedicated toward laying the foundations for a modern theory of demand.

Indeed, we saw in the preceding section that Pareto used to analyze independent and symmetric substitutes on the one side, and asymmetric substitutes on the other. Johnson will change this dichotomy into another one, considering that substitutability is in fact interesting only to the extent that it reflects a hierarchy of preferences. Johnson names "demand curves" the two kinds of curves obtained by considering either prices as constant and varying income ("varying expenditure *curve*" (Johnson, 1913, 103)) or a change in relative prices ("varying *price* curve" (Johnson, 1913, 103)). Johnson's aim is to discuss the shape of those "demand curves" and to link them with the shape of indifference maps. Johnson's contribution is however curious, because there is no reference to earlier contributions. Pareto, Fisher, Auspitz and Lieben are not mentioned in the article.[39] It is one more "missed appointment" of the history of demand and choice. So

[39] Johnson is very thrifty in his comments on the analytical developments of his article, and he is patently reluctant to establish some connections with other contributions to the same topic. Thus Edgeworth is the only writer to whom he refers in the article. That there is no other reference to Auspitz and Lieben, Pareto, or Fisher, is rather curious. With respect to Edgeworth, Johnson does not even borrow the terminology of "indifference lines" or "indifference curves", his preferred "own" terminology being that of "constant utility curve". In the same spirit, the systematic use of the ratio of marginal utilities could be linked to Fisher or Pareto. It is therefore difficult to believe that Johnson could have not been acquainted with all those writers. For instance, some passages in the article do present a striking resemblance to Auspitz and Lieben (1889). Knowing that Lieben had published an article on consumer's surplus in the *Economic Journal* (Lieben, 1894), it is reasonable to think that he could have been led to their book through this article. Moreover, knowing that Johnson had written an article with Sanger (Johnson and Sanger, 1894), and that Sanger was to publish a review article on the developments of mathematical economics in Italy the next year, in which Pareto is treated at length (Sanger, 1895), it is very doubtful that Johnson did not become acquainted with Pareto's work. All this, of course, remains pure speculation.

Johnson's article provides a clarification and development of Pareto's insights, although maybe without being acquainted with Pareto's work:

> "In Part II [of the article] the analytical conditions which govern the shape of the constant utility curves are elucidated. Criteria are given for distinguishing between three types of the resulting demand curves and precise definitions are introduced for the terms "complementary" and "competitive" as applied to commodities" (Johnson, 1913 (1968), 97)

Moreover, it seems reasonable to say that the distinction between two kinds of "demand curves" is a device, if only still imperfect, to analyze the Giffen case, and more broadly, the law of demand:

> "In Part III [of the article] (…) it is found possible to analyze the case in which increased total expenditure *diminishes* the amount bought of one of the commodities, as well as the standard case in which more of both commodities is bought, when there is more to spend. Similarly, when the demand curve [varying price curve] is reached, for which one of the commodities varies in price while the total expenditure is fixed, we can analyze the case in which an increased price leads to an *increase* of the amount of the commodity bought." (Johnson, 1913 (1968), 97-98)

And again, that Johnson captured the importance of the expenditure curve for the law of demand is clear from this:

> "The character of the varying expenditure curve will be considered at length in Part II. From it the characteristics of the varying price curve can be conveniently deduced" (Johnson, 1913 (1968), 103)

Diagrammatically, the quintessence of Johnson's discussion is contained in the fig.4 of the article, presenting the shape of the "varying expenditure *curve*" and of the "varying *price* curve".

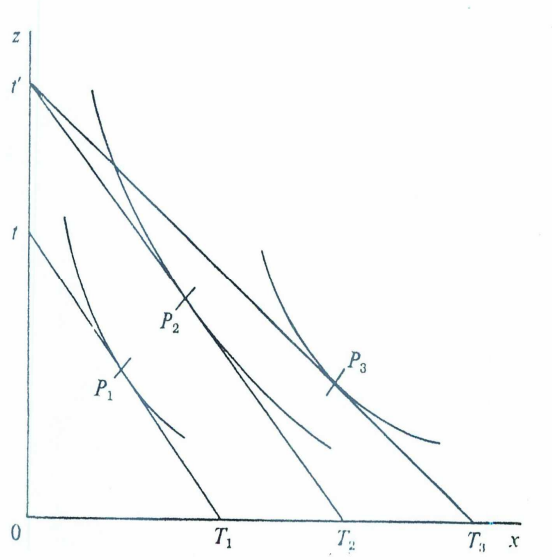

Fig.4: Johnson's price effect and income effect.
(Source: Johnson, 1913, 102)

Johnson considers two marginal rates of substitution, either relatively to *x* or relatively to *y*. With $u = \varphi(x, y)$, noting R_y^x the marginal rate of substitution of *y* to *x* (a notation that is not in Johnson's article) (Johnson, 1913, 107):

Code de champ modifié

Code de champ modifié

(10) $$R_y^x = \frac{\varphi'_x}{\varphi'_y} \text{ and } R_x^y = \frac{\varphi'_y}{\varphi'_x} \qquad R_y^x . R_x^y = 1$$

Johnson will also base his discussion on some properties of the utility function.

In the "standard case", he supposes that (Johnson, 1913, 105)

(11) $$\frac{-\varphi''_{x^2}}{(\varphi'_x)^2} > \frac{-\varphi''_{xy}}{(\varphi'_x.\varphi'_y)} \text{ and } \frac{-\varphi''_{y^2}}{(\varphi'_y)^2} > \frac{-\varphi''_{xy}}{(\varphi'_x.\varphi'_y)}$$

The interpretation of this "standard case" is that "a change in the amount of *x* would produce a greater relative change in the marginal utility of *x* than in that of *y*" and respectively "a change in *y* would produce a greater relative change in the marginal utility of *y* than in that of *x*" (Johnson, 1903, 105).[40]

In the non-standard case, it is possible that "a change in *y*, as well as a change in *x*, produces a greater relative change in the marginal utility of *x* than in that of *y*". Mathematically, this gives the following system (Johnson, 1913, 105)

(12) $$\frac{-\varphi''_{x^2}}{(\varphi'_x)^2} > \frac{-\varphi''_{xy}}{(\varphi'_x.\varphi'_y)} \text{ and } \frac{-\varphi''_{y^2}}{(\varphi'_y)^2} < \frac{-\varphi''_{xy}}{(\varphi'_x.\varphi'_y)}$$

The symmetrical system is also possible, but the possibility that both *x* and *y* produce a greater relative change on the marginal utility of the other good than on their own marginal utility is ruled out. And in case (12), Johnson imposes a supplementary restriction: "although a change in *y* as well as a change in *x* may produce a greater relative change in the marginal utility of *x* than in that of *y*, yet such excess as is due to a change in *x* (measured relatively to the marginal utility of *x*) will invariably be greater than such excess as is due to a change in *y* (measured relatively to the marginal utility of *y*)" (Johnson, 1913, 105). Mathematically, this comes to the additional constraint to inequality (12):

(13) $$\frac{-\varphi''_{x^2}}{(\varphi'_x)^2} - \frac{-\varphi''_{xy}}{(\varphi'_x.\varphi'_y)} > \frac{-\varphi''_{xy}}{(\varphi'_x.\varphi'_y)} - \frac{-\varphi''_{y^2}}{(\varphi'_y)^2} > 0$$

The next step in the classification of indifference maps is to study the sign of $\frac{dR_y^x}{dx}$ and that of $\frac{dR_y^x}{dy}$, using the inequalities (11), (12) and (13).

Johnson shows that $\frac{dR_y^x}{dx}$ has the sign of $\frac{-\varphi''_{xy}}{(\varphi'_x.\varphi'_y)} - \frac{-\varphi''_{x^2}}{(\varphi'_x)^2}$ and that $\frac{dR_x^y}{dy}$ has the sign of $\frac{-\varphi''_{xy}}{(\varphi'_x.\varphi'_y)} - \frac{-\varphi''_{y^2}}{(\varphi'_y)^2}$ (Johnson, 1913, 107)

[40] This kind of assumption is strongly related with the idea of decreasing marginal utility, and so Johnson's emancipation from cardinalism should not be taken for granted.

From this, Johnson goes on directly to produce a typology of the three possible local properties ("section") of an indifference map ("adjacent utility curve")

(14a) In the standard or mediate section, $\frac{dR_y^x}{dx} < 0$ and $\frac{dR_x^y}{dy} < 0$

(14b) In the *y*-urgent section, $\frac{dR_y^x}{dx} < 0$ and $\frac{dR_x^y}{dy} > 0$

(14c) In the *x*-urgent section, $\frac{dR_y^x}{dx} > 0$ and $\frac{dR_x^y}{dy} < 0$ (derived from (12) when *y* and *x* are inverted)

Moreover, (13) prevents dR_y^x/dx and dR_x^y/dy from being simultaneously positive.

After pointing out the "extreme cases" when indifference lines are at right angle or linear (the case of perfect complements and perfect substitutes introduced by Pareto), Johnson suggests that the curvature of an indifference curve may be taken for an indicator that the goods *x* and *y* are "roughly competitive or complementary" (Johnson, 1913, 107), Johnson immediately corrects this momentary slip and introduces "a more exact definition of competitive and complementary" (Johnson, 1913, 107).

From (13), it follows that:

(15) $$2\frac{\varphi''_{xy}}{(\varphi'_x.\varphi'_y)} - \frac{\varphi''_{y^2}}{(\varphi'_y)^2} - \frac{\varphi''_{x^2}}{(\varphi'_x)^2} > 0$$

which is equivalent to:

(16) $$2\varphi''_{xy} > \varphi''_{y^2}R_y^x + \varphi''_{x^2}R_x^y$$

Johnson's contribution is based on a classification of the relations among goods according to the sign of the derivatives of the marginal rates of substitution. Thus, taking the cross second-order derivative of the utility function as the implicit definition of complementary and competitive goods, Johnson proposes another definition depending on the value of φ''_{xy} relatively to the value of $\varphi''_{y^2}R_y^x$ and $\varphi''_{x^2}R_x^y$. To sum up:

(17a) *x* and *y* are competitive if: $\varphi''_{y^2}R_y^x + \varphi''_{x^2}R_x^y > \varphi''_{xy} > \frac{\varphi''_{y^2}R_y^x + \varphi''_{x^2}R_x^y}{2}$

(17b) *x* and *y* are complementary if: $\varphi''_{xy} > \varphi''_{y^2}R_y^x + \varphi''_{x^2}R_x^y$

From this presentation of Johnson's argument, Johnson's definition of complementarity turns out to be a reformulation, in terms of utility, of the distinction previously proposed between the standard-mediate case and the *x*- or *y*-urgent cases. Indeed, the sign conditions on dR_y^x/dx and on dR_x^y/dy can be rewritten:

Code de champ modifié

(18) dR_y^x/dx has the sign of $\varphi'_y\varphi''_{x^2} - \varphi'_x\varphi''_{xy}$

(19) dR_x^y/dy has the sign of $\varphi'_x\varphi''_{y^2} - \varphi'_y\varphi''_{xy}$

From $\varphi'(.) > 0$ it follows that both dR_y^x/dx and dR_x^y/dy are positive whenever $\varphi''_{xy} > 0$ but also when $\varphi''_{xy} < 0$ is small enough (in absolute value). The question is thus to express the values φ''_{xy} at which (18) and (19) are nil. Those values are respectively $\varphi''_{x^2} R_x^y$ and $\varphi''_{y^2} R_y^x$.

Then, whenever $\varphi''_{xy} < \max\left\{\varphi''_{x^2} R_x^y, \varphi''_{y^2} R_y^x\right\}$ either dR_y^x/dx or dR_x^y/dy will be positive and the i-urgent case prevails. Changing the name "mediate" and "urgent" for "complementary" and "competitive", one obtains directly (17a) and (17b).

In the final analysis, Johnson's definition is entirely based on the shape of indifference maps. His reference to the value of the cross second-order derivative of the utility function shows that he did not ignore this criterion, and that he was interested in finding some relations between the utility based approach and the preference-based approach. But from his definition, the limit value of φ''_{xy} between complements and substitutes is always negative, and above all, it is no longer constant.

From this expository of Johnson's recasting of complementarity, two partial conclusions are in order. Firstly, Johnson's shift from Pareto's *Manual* was certainly essential for Hicks and Allen subsequent break with the Auspitz-Lieben-Edgeworth-Pareto criterion. Fundamentally, the inequalities (17a) and (17b) are not proposed as definitions of complementarity. They are just exhibiting that there is almost no overlap between the new criterion and the old one. Secondly, Johnson's break is also sensible, at least negatively, from his poorly motivated own definition, which is just a convenient way to make a distinction between the mediate case and the urgent case, and beyond, to develop analytically relevant tools for the law of demand.

3. Substitutability and consumer surplus

From what has been said in the two previous sections, it is necessary to understand what kind of difficulties the concept of substitutability was aimed at working out. Since Fisher (1892), the search for complementarity is above all oriented toward improving demand theory or price theory. But it was not necessarily the case before. Indeed, Auspitz and Lieben (1889) on the one side and Marshall (1890) on the other all had in mind to provide a useful and precise measure of consumer's surplus. Historically, the case of complements and substitutes may well be coextensive to the concept of consumer surplus. In 1847, a French engineer, Louis Bordas challenged Dupuit's calculation of a surplus value in his 1844 memoir ("De la mesure de l'utilité des travaux publics"):

> "Let us suppose that we have got to evaluate the utility of one kilogram of meat and that we ask to a person what sacrifice he would be likely to do in order to get this kilogram. Can we expect this person to give a straight answer? Of course we cannot. Is it not depending on the wealth of that person and in the same time on the current prices of other

foodstuffs that may be substituted for meat? If potatoes are cheaper than meat, the maximum sacrifice that the poor will endure will be less than that for potatoes; in the opposite situation, it will be higher. Indeed, he will prefer to live the whole year with potatoes rather than to have a good meal for a few days an then starve to death. Thus, the maximum sacrifice for meat is influenced by the current prices of potatoes and other foodstuffs. *What theory can be erected on such a variable basis, depending both on taste and wealth of each consumer?*" (Bordas, 1847, 92, italics ours)

Otherwise stated, Bordas underlines the difficulty for a theory of consumer surplus to isolate the tastes from the needs. Indeed, the question here is whether substitutes will be available at a lower price, that is if substitutability is possible not only formally but practically. This is of crucial importance, for the choice of poor people may take place not far from the subsistence level. Similar difficulties will be at the forefront of with Marshall and with Auspitz and Lieben studies.

3.1. Marshall's consumer surplus and complementarity

In taking substitutes seriously, Marshall's aim is not to lay the foundations for a better justification of the law of demand. His aim is to improve the analysis of consumer's surplus and consumers' surplus. So, what is at stake when he deals with complementarity is the legitimacy of a consumer's surplus index. This is certainly a key for reading chapter 3 of book III of the *Principles* dealing with the law of demand, and Aldrich (1996, 171) has rightly characterized surplus theory in Marshall's work as the "raison d'être" of the theory of demand. Theory of demand in the *Principles of Economics* is designed to serve as a basis for analyzing consumer surplus.

When he comes to the circumstances affecting the demand curve, Marshall does not miss the impact in the change of the price of a substitute good:

"The demand prices in our list are those at which various quantities of a thing can be sold in a market *during a given time and under given conditions*. If the conditions vary in any respect the prices will probably require to be changed; and this has constantly to be done when the desire for anything is materially altered by a variation of custom or by *a cheapening of the supply of a rival commodity* [italics ours], or by the invention of a new one. For instance, the list of demand prices for tea is drawn out on the assumption that the price of coffee is known; but a failure of the coffee harvest would raise the prices for tea. The demand for gas is liable to be reduced by an improvement in electric lighting; and in the same way a fall in the price of a particular kind of tea may cause it to be substituted for an inferior but cheaper variety." (Marshall, 1890 (1898), book III, ch.3, 174-75)

Otherwise stated, Marshall suggests that the variation of the price of a good has an effect on the demand curve for a substitute. Implicitly, this is a difficulty for constructing a demand schedule that will be useful for

consumer surplus. He also considers (Marshall, 1890 (1898), book III, chap.3, §6, 175, n1)[41] the case of a group of different qualities of tea that are affected by a common decrease of their price (without change in their relative price), so that the relative price of tea has decreased. At an aggregated level, he recognizes that the relative decrease in the price of tea should induce a global increase in the consumption for tea and in the same time that it might induce a decrease in the quantity consumed of inferior varieties of tea, although not necessarily. Of course, this example has nothing to do with a law of demand, considering that other prices (of different varieties of tea) did not remain constant.

This example leads to a pragmatic question: How should one group different goods when one wants to derive a law of demand for those goods? Marshall simply argues that it is a question of convenience. On many occasions, he puts this into practice (Marshall, 1898, 180, n.1). This leads to a contradiction in the Marshallian theory of demand. Marshall's attitude is always oscillating between the unrealistic assumption that the price of a good is independent of the price of its substitutes and the opposite assumption that the relative prices of substitutes for a good are fixed, so that all substitutes can be taken as a single composite good.

What can be drawn from this pragmatic and schizophrenic attitude? That Marshall wishes to take away all the difficulties linked with the measure of the value in use for a good, independently of other goods. All this theoretical neutralization of substitutes is linked with the goal of achieving a measure of consumer surplus.

In the *Principles*, Marshall devotes many pages to constructing a measure of surplus with special attention to the ways to neutralize substitutes and complements, and that leads to an additively separable utility function.

In many occurrences, Marshall's aim is to estimate the variation of surplus consecutive to a slight variation of the price of a good (Marshall, 1898, 208, n.1). The measure of consumer surplus supposes first to compare the marginal utility of money for different persons (Marshall, 1898, 206). But those are not the only conditions, at least if the measure of surplus does not have any sense by itself but only through a comparison with other measurements of surplus.

As has been said, Marshall takes an additive utility function as a good representation of total utility. This way of representing utility is not without weakness if the demand schedule must serve to the measurement of the surplus. In case of goods that have the same use, the measure of surplus of a good x is not yet independent of the surplus measure of a good y, and the *ceteris paribus* condition for the derivation of a demand schedule is not verified, even approximately:

> "It will be noted however that the demand prices of each commodity, on which our estimates of its total utility and consumer's surplus are based,

[41] "It is even conceivable, though not probable, that a simultaneous and proportionate fall in the price of all teas may diminish the demand for some particular kind of it; if it happens that those whom the increased cheapness of tea leads to substitute a superior kind for it are more numerous than those who are led to take it in the place of an inferior kind"(Marshall, 1890 (1898), book III, chap.3, §6, 175, n1)

assume that other things remain equal, while its price rises to scarcity value: and when the total utilities of two commodities which contribute to the same purpose are calculated on this plan, we cannot say that the total utility of the two together is equal to the sum of the total utilities of each separately." (Marshall, 1898, 206)

What can be drawn from all this is that the measurement of consumer surplus cannot dispense with interdependences of marginal utilities. But from this point of view, the theoretical treatment is very similar for substitutes and complements, and it would be exaggerated to speak of a concept of complementarity. This solution underlines with more intensity that Marshall's notion of complementarity is still in embryo:[42] Even if Marshall's typology remains approximate, it plays a specific role in the *Principles* at the juncture between demand and consumer's surplus. From this point of view, Marshall did not need to develop a much richer and refined concept. He just tried to make precise the conditions for applying surplus measurement. Accordingly, complementarity does not reach the conceptual level with Marshall.

Shortly after, in the *Principles*, this unsatisfactory construction will be staggered by the Giffen case. Indeed, when he introduces the Giffen case (Book III, chapter 6), Marshall would create new difficulties for the theory of consumer surplus. Whereas the grouping of competitive goods was developed in order to justify the use of an additive utility function, the Giffen case puts into the foreground the special case for which the marginal utility of expenditure is no longer constant. Marshall's point is not that the demand schedule might be increasing (this is just a possible after-effect) but that the marginal valuation of money might change, and consequently that it might jeopardize the theoretical conditions for an appropriate measurement of consumer surplus. To that extent, the Giffen case can be interpreted as a special kind of situation where many substitutes are at stake but cannot be grouped together, because the substitution that would allow for a common variation of the price of the many substitutes would be incompatible with a constraint on the subsistence level.[43] And Marshall's discussion is aimed at marginalizing the occurrences of this situation and at asserting the possibility of a measurement of surplus.

In the final analysis, neither the *ceteris paribus* condition, nor the grouping of goods seems to be a reasonable solution. The variation in the marginal utility of expenditure is only a synthetic sign of this. For this reason, it is usually underlined that Giffen goods occur when the good whose price is raised cannot be substituted with other goods at a cheaper price (Marrewij and Bergeijk, 1990), and that the expenditure for this good and its substitutes is important. The presence of substitutes is thus one among other conditions for the Giffen case. The conclusion to be drawn

[42] "the desire for anything is much dependent on the difficulty of getting substitutes for it." (Marshall, 1898, book III, ch.6, 208). See also Stigler (1950 (1965), 106-107)

[43] For a different point of view, see Silberberg and Walker (1984, especially page 693). They develop the analysis of Giffen goods within the context of an additive utility function, *but* with variable marginal utility of income. See also Stigler (1947), Dougan (1982) and Dooley (1992)

from this is that Marshall did not develop a concept of complementarity, except in a defensive way in consumer surplus arguments. Comparatively, Auspitz and Lieben's strategy may appear as offensive.

3.2. Auspitz and Lieben's substitution principle

The implementation of a theory of consumer surplus is certainly the main objective of Auspitz and Lieben's *Investigations*. Accordingly, as for Marshall in the *Principles*, demand theory is just a step towards a theory of consumer surplus.[44]

The leading idea of the *Investigations* is to compare the total surplus derived from the consumption of two bundles, i.e. to compare in each case the difference between the willingness to pay for this bundle and the actual expenditure on it. Marginal utility of money being constant, the increase in satisfaction "corresponds exactly to that sum of money" (Auspitz and Lieben, 91). And thus, Auspitz and Lieben infer that any individual

> "will be in a position to measure, in money terms, the difference in satisfaction resulting from different combinations of consumption for given prices of all articles." (Auspitz and Lieben, 1889 (1914), 92)

Thus, Auspitz and Lieben seem to accept the idea that money is an extensive measure of utility, provided that the marginal utility of money is constant. The conception of a utility measurement as a marginal valuation is not far from Jevons's attitude, once a non additive separable utility function is assumed.[45] A few years later, Lieben will take a part in the Edgeworth-Nicholson debate on consumer rent,[46] and he will specify that they (Auspitz and Lieben) had examined "maturely the adequacy of such a measure" (Lieben, 1894, 716):

> "We measured the difference of utility attributed by the same individual to two different objects or combinations of such, by the surplus payment

[44] This subsection borrows amply from Lenfant (2000)

[45] "We can seldom or never affirm that one pleasure is an exact multiple of another; but the reader who carefully examines the following theory will find that it seldom involves the comparison of quantities of feeling differing much in amount. The theory turns upon those critical points where pleasure are nearly, if not quite, equal. I never attempt to estimate the whole pleasure gained by purchasing a commodity; the theory merely expresses that, when a man has purchased enough, he would derive equal pleasure from the possession of a small quantity more as he would from the money price of it" (Jevons, 1871 (1965), 13). According to Houghton (1958, 54), Auspitz and Lieben's conception of utility measurement looks like an "highly sophisticated Jevonian utilitarianism".

[46] This debate was initiated by Nicholson in his *Principles of Political Economy* (1893), criticizing Marshall's theory of consumer's surplus. Edgeworth took Marshall's defense in 1894 in the *Economic Journal*. Nicholson answered in the same review, focussing mainly on the meaning and justification of a constant marginal utility of money (see Maloney, 1985, 77-79). The debate is a controversy over the merits of consumer's surplus analysis. Nicholson did not adhere to the idea of measuring utility. According to Nicholson, it is not possible to dispense with the variations in the marginal utility of money. Restricting "the application of consumer surplus analysis so that necessaries are not involved" (Maloney, 1985, 79) is still not enough. He concludes that the theory would be applicable "to a few careless millionaires." (Nicholson, 1894, quoted in Maloney, 1985, 79). His criticism was influential on the third edition of Marshall's *Principles*.

of money necessary to render the said individual indifferent to the choice of the one or the other. We never stood in need for what different individuals feel (...). We compare enjoyment with enjoyment, and at the point of equilibrium, when the utility of a certain quantity of an article is just sufficient to determine the purchaser to spend the money required, this sum gives an exact measure of the utility whatever may be the motives." (Lieben, 1894, 717)

Thus, consumer's surplus is conceivable in the neighborhood of equilibrium, when individuals react to small variations in prices. Consequently, utility and demand analysis should be analyzed in that context.

On many occasions, Auspitz and Lieben do insist on the idea that consumers can easily find substitutes in case of the price increase of a good. A first instance of this is when they take account for the measure of consumer surplus when goods are necessary or subsistence goods. The measure of consumer surplus is only possible as long as it does not imply some measurement for the utility provided by indispensable quantities. Actually, the main difficulty raised by necessary and indispensable goods is that the marginal utility of money is no longer constant, and that in those cases, the surplus measurement cannot be balanced with other variations of surplus that would not induce a similar change in the marginal utility of money. As a reason for taking this for granted and for reducing the range of situations that would induce a variable marginal utility of money, Auspitz and Lieben note that:

"The air necessary to breathing being expected, no article exists which is indispensable or which is irreplaceable by some substitutes." (Auspitz and Lieben, 1889 (1914), 105)

And later,

"the different kinds of beverages can be replaced and substituted to one another, following slight changes of prices. Reciprocal relations of the same type do exist in every group of articles satisfying analogous needs." (Auspitz and Lieben, 1889 (1914), 110)

Lieben's contribution to the Edgeworth / Nicholson debate on consumer's surplus in 1894 was the occasion for bringing the originality of their analysis to light: [47]

"For any individual and for any definite quantity of a certain article, a sum of money is imaginable which represents the maximum he is willing

[47] Lieben's short 1894 article is an answer to one of Nicholson's arguments against utility measurement. An exposition of the debate can be found in Maloney (1985, 79). The debate is a controversy over the merits of consumer's surplus analysis. Nicholson did not adhere to the marginal utility theory and even less to the idea of measuring such an intangible object as utility. According to Nicholson, it is impossible to dispense with the variation in the marginal utility of income. Restricting "the application of consumer surplus analysis so that necessaries are not involved" (Maloney, 1985, 79) is still not enough. He concludes that the theory would be applicable solely "to a few careless millionaires" (Nicholson, 1894, quoted in Maloney, 1985, 79).

to expend for it. Rather than pay more, the individual in question would renounce the acquisition and enjoyment of the quantity in question. Substitutes, extraordinary cases excepted, being always at hand, this forbearance does not involve the absolute renunciation of the enjoyment to be provided by the said article, an analogous enjoyment by means of a substitute being available at its fixed price." (Lieben, 1894, 718)

Thus, substitutes, taken as a relevant feature of the economic world, are involved in a defense of consumer surplus, and the case for a variable marginal utility of money should be restrained. For a given income, an individual faced with a rise in he price of a good may always – almost always – be able to find substitutes that will satisfy the same need, and thus to choose a different bundle without being subjected to a variation of the marginal utility of income. Of course, this does not rule out Giffen cases.

4. Complementarity and demand theory

As was pointed out in the introduction, complementarity is often presented and sometimes defined within price-quantity relations, that is, within empirical and observational relations. This way of introducing complementarity directly at the market level is prior to the utility-based treatment.

Once the utility-based or preference-based approach were developing, this empirical point did not stand as a rival typology, but rather as a set of logical consequences of the utility- or preference-based approaches. For instance, the need for a demand-based approach was implicitly underlined by Walras in his *Eléments d'économie politique pure* when analyzing the tâtonnement process, and also in the 15th lesson. It remains that for the vast majority of the writers, a demand-based approach should be based on a utility based definition, with reference both to subjectivity and to well identified given needs

The aim of this sub-section is to bring to the fore the ways and means of an instrumentalization of complementarity in relation to the theory of demand and price theory. It will be convenient first to oppose Auspitz and Lieben, on the one hand, and Fisher on the other hand, both works being centered on price theory. Then, we will turn to Pareto and Johnson's treatment of the law of demand. With Pareto, and to a lesser degree with Johnson, mathematical economics has been progressively mould to form a basis empirical testing and observational theorems, and substitutability will be torn between two sides: the empirical side and the introspective side.

Paradoxically as it may seem, Pareto pushed the utility-based concept as far as possible for demand theory, and he simultaneously revealed that the utility based concept was not powerful and useful for demand theory, because it was not enough to derive significant conclusions on demand theory.

4.1. The empirical content of complementarity, according to Auspitz, Lieben and Fisher.

The theoretical use of complementarity differs between Auspitz-Lieben and Fisher. Whereas the Auspitz and Lieben emphasis is on the variation of the optimal bundle for the consumer, Fisher will focus on the properties of the price system. Whereas Auspitz and Lieben will consider the effect of a price variation on the chosen bundle for one individual (other prices being fixed), Fisher claims to adopt a general equilibrium framework, and he will suggest a direct relationship between complementarity and the variations of relative prices.

In Auspitz and Lieben's *Investigations*, substitutes and complements are involved in two respects in demand analysis. On the one hand, the authors will focus on what may be called "related demands", that is, on the study of the interdependencies between demands for different goods according to whether those goods are substitutes or complements to each other. Besides, it is in that context that the definition of substitutes and complements is first introduced. On the other hand, they use substitutes in order to justify the law of demand.

Let us begin with the analysis of related demands. In the main part of the *Investigations*, Auspitz and Lieben introduce the notions of competing goods and complementary goods to provide a more complete analysis of individual demand behavior:

> "If, for the price of the article (x), we make successively different assumptions, the quantity consumed of this article will vary, and every time it will be associated with other combinations of advantageous consumption, with different quantities of the other articles. For many of them, there will be no alteration; for some of them, the alteration will be in the same direction as for the article (x), and for the others, it will be in the opposite direction. In general, the increase in the consumption of the article (x) will induce a rise in the consumption of those articles which are used to *complete* the enjoyment expected from the consumption of (x); thus, the consumption of sugar increases with that of coffee, of tea, and so on. On the contrary, if those articles are *competing* with (x), their consumption will usually diminish with an increase in the consumption of (x): it is the case of coffee and tea, of wine and beer and brandy, and so on." (Auspitz and Lieben, 1889 (1914), 97-98)

Thus, the introduction of substitutes and complements is the starting point of a thorough study on related demands. It is a way to appraise the changes in the most advantageous bundle associated to the variation in the consumption of one good. Nevertheless, the authors are quite cautious regarding demand for substitutes and complements. Indeed, the "general" law mentioned above does not rule out some peculiar cases:

> "It is nevertheless possible that some opposite manifestations may occur, because of the many cross-linked influences between goods. Thus, a rise in the consumption of coffee will always result in an increase in the quantity of sugar used to sweeten it, but, if the same individual is also reducing his consumption of tea in consequence of an increased use for coffee, it may happen that, instead of increasing, its total consumption of sugar should diminish." (Auspitz and Lieben, 1889 (1914), 98).

Thus, excepted for peculiar case, Auspitz and Lieben do adopt a law of related demands. According to this law, whenever two goods *i* and *j* are complements to each other, and $dx_i/dp_j < 0$. Exceptions to this general rule may take place, once it is taken account for complex interrelations in the more than two goods case.[48] As far as I know, Auspitz and Lieben were the first to provide this example. But nowhere do they discuss these properties of related demands in the light of the Auspitz-Lieben utility criterion.

Code de champ modifié

Let's come now to the law of demand.[49] Auspitz and Lieben do not provide a justification for the individual law of demand, in the context of the more general representation of utility. But when they expose the law of market demand, they refer to a substitution principle, identical with Marshall's early justification.[50] It is quite obvious that for a given good, some individuals will not buy it as long as the price of the good is too high. As a consequence, it may happen that for some high prices, there is a zero demand for some individuals. This "non effective demand" represents the demand of those consumers who, because of the current price,

> "drop the idea of acquiring the article in question and can be contented with a substitute, but that would be effective buyers on the market with a lower price." (Auspitz and Lieben, 1889 (1914), 24-25)

Once more, we find here the general hypothesis about substitutability in the economic world. But in the same time, a new assumption surfaces: that consumers are likely to find substitutes at lower prices. Thus, the law of demand is implicitly founded on the idea that each consumer is likely to find substitutes at a lower price.

As for Fisher (1892), the use of complementarity is analogous with that of Auspitz and Lieben. I mean that both works contain disseminated remarks and constructions appealing with more or less ostentation – and conviction – to complementarity.

From his analysis of the interdependencies of marginal utility (seen above), Fisher draws a conclusion as to the general observable properties of substitutes and complements:

> "The essential quality of substitutes or competing articles is that the marginal utilities or the prices of the quantities actually produced and consumed tend to maintain a constant ratio. We may define *perfect substitutes as such that this ratio is absolutely constant.* The essential attribute of completing articles is that the ratio of the *quantities* actually produced and consumed tends to be constant (as many shoe-strings as shoes for instance, irrespective of cost). We may define *perfect*

[48] Fisher (1892, 77-8) will underline a similar complexity regarding indifference surfaces in the at least three-goods-case. This is another proof that Auspitz and Lieben's *Investigations* were of influence to Fisher (1892).

[49] Chipman (1977) has shown that if all distinct commodities are ALEP complements of each other, then the law of demand must hold: income elasticities of demand must be positive and demand for a commodity must be a decreasing function of its own price.

[50] The justification is exactly that of Marshall in *The Pure Theory of Domestic Values*. But Marshall's essay was published in 1879 "for private circulation", and the authors did not know about it (see Auspitz and Lieben (1889 (1914), xiv-xv)).

completing articles as such that this ratio is absolutely constant." (Fisher, 1892, 66).

Leaving aside considerations on demand, the preceding quotations suggest the following comment. Fisher proposes an analysis in terms of quantifiable and observable relations (price-quantity) as a polar system to classify relations between goods. If Fisher's argument is read carefully, this polar system is not a new definition of substitutes and complements, but rather an empirical implication of those properties when goods are perfect substitutes or complements in terms of utility. But nowhere does he provide an analytical definition of perfect substitutes and complements. Fisher does not indicate what are the links between those polar cases and the definitions in terms of utility.

Supprimé : and some others before 1915

4.2. Toward a turning-point: Pareto

With Pareto, a shift will take place regarding complementarity and demand. The conceptualization of complementarity, its homogeneity, will develop within the framework of a theory of choice. To this respect, the rationalization of complementarity is on a par with the need for analytical tools in demand theory, and the reader will find only little appeal to proprieties of the price structure or of the choice structure that do not fit narrowly with the definition of complementarity.

It must be clear that Pareto's introduction of complementarity is linked in all respects to a dissatisfaction with the current economic theory of demand based on additive utility functions.[51] The theoretical implications of additivity go against the facts that not all goods are superior goods, and that, on the contrary, the existence of many different qualities induces on many occasions that a rise in the budget will be accompanied by a decrease in the quantity consumed of inferior qualities.[52]

Supprimé : substitutability

To put it briefly, it is well known that Pareto came most of the way in deriving the Slutsky equation in the October 1893 issue of the *Giornale degli economisti* (Chipman, 1976; Dooley, 1983; Weber, 1999). In the course of his theorizing about demand, Pareto will give the most general expression of the time regarding the effect of a price change on the demand for a good. As a comment to this expression (equation (75) in Pareto, 1909 (1971), 423), Pareto will underline both the possible implication of complementarity in the Auspitz-Lieben sense, but he will also remark that

[51] The evolution of Pareto is very sensible from the *Cours* to the *Manual*. In the *Cours*, Pareto settles the issue of related demands for complements by considering a composite commodity, and the case of substitutes is ruled out by considering that the quantities consumed will be "linked by certain equations" (Pareto, 1897, 322n).

[52] "All this s in obvious contradiction to the facts (…) since for a very large number of goods there are a certain number of grades of each. And, as income increases, the superior grades replace the inferior grades; consequently the demand for these latter increases at first with the increase in income, but then decreases until it becomes insignificant or even nil" (Pareto, 1909 (1971), 198)

this criterion cannot by itself serve as a basis for the law of demand, except in the case of complementary or independent goods:

> "Let us consider supply and demand with reference to one individual who has two or more goods at his disposal. If the consumption of these goods is independent, or if there is a dependence of the first type between them, the demand for a good always decreases with an increase in the price of that good; (…) For goods having a dependence of the second type, when the price rises, the demand may increase and then decrease" (Pareto, 1909 (1971), 197-198)

From this comment, we can find some anticipations of the Johnson criterion of complementarity, in case of structural relationships of independence or complementarity.

Accordingly to this lack of empirical implications of complementarity, Pareto shows that it is not easy to develop a solid basis for demand theory on the basis of complementarity only. And this is precisely the reason why he insisted so much on the relations between income and demand. This intermediate construction, as he sees it, his perfectly adapted to empirical examination:

> "In real life an individual generally demands a great variety of goods and supplies only one or a few. A very great number supply only labor; others, the use of savings; others, certain goods which they produce. The case of simple exchange of two goods between which there is a dependence of the second type is definitely the exception; a workman sells his labor and buys some corn and bread, but we do not observe the exchange of bread for corn. Hence the deductions of the theory could not be directly verified in this case, and we would need to have another procedure for verification, which we can get by considering the allocation of income" (Pareto, 1909 (1971), 199)

What emerges from the *Manual* is in fact that Pareto recognizes the individual's income as an essential variable for demand theory. This is not new, compared with Fisher and others, but Pareto's reflection is more clearly linked to the idea of studying market demand, and not only individual demand:

> "We speak of the individual, not in order actually to investigate what one individual consumes or produces, but only to consider one of the elements of a collectivity and then add up the consumption and the production of a large number of individuals" (Pareto, 1909 (1971), 123)

Instead of focusing solely on the opportunity to find substitutes, Pareto also stresses the role of income. It follows logically that complementarity will be therefore only one among other explanatory variables for demand analysis.

As a conclusion to this, Pareto appears to have exhibited in all respect the empirical significance of complementarity, and to have learned from this the need for supplementary tools in order to study demand behavior and to give it an empirical flavor. This is both theoretically and methodologically a step towards modern consumer theory.

5. Slutsky and complementarity: the mind-body problem

Here we arrive at least at the crux of the matter, at the acme in the construction of the concept of complementarity. As was said in the introduction, Slutsky's 1915 article allowed Schultz and Hicks to develop a modern, empirically fruitful definition of complementarity, on the basis of the Slutsky equation, but Slutsky himself, although he was certainly interested in complementarity, did not elaborate on his own equation. The purpose of this section is to investigate Slutsky's article in order to suggest some explanations for this.

To put it in a few words, I would like to interpret Slutsky's article as an exacerbation of Pareto's methodological eclecticism or dilemna. Thus I would like to interpret Slutsky's silences on complementarity as an unwillingness to outdo the theoretical inconsistencies of the Paretian heritage. Let us come back first to the most well-known outcome of Slutsky's article, dealing with the formal aspects of the theory of choice.

5.1 The rejection of the Auspitz and Lieben criterion

Slutsky's analysis of demand is in the spirit of the Paretian theory of the equilibrium of the individual consumer, and the analytical results reached by Slutsky can be interpreted as the culmination of the Paretian program (Dooley, 1983; Weber, 1999)

Slutsky's famous equation is expressing the variation in the demand of a good in reaction to a variation of the price of another good. Adopting Slutsky's notation:

(20) $$\frac{\partial x_i}{\partial p_j} = u' \frac{M_{ij}}{M} - x_j \frac{\partial x_i}{\partial s}$$

Code de champ modifié

where x_i is the quantity demanded of commodity *i*, p_j is the price of commodity *j*, and *s* is income, while *u'* is the marginal utility of income, *M* is the determinant of the matrix of second-order partial derivatives u_{ij} of the utility function bordered by the prices, and M_{ij} is the minor of this matrix corresponding to the element u_{ij}.

Slutsky also derived the following symmetry "condition" (equation [55])

(21) $$k_{ij} = u' \frac{M_{ij}}{M} = \frac{\partial x_j}{\partial p_i} + x_i \frac{\partial x_j}{\partial s} = \frac{\partial x_i}{\partial p_j} + x_j \frac{\partial x_i}{\partial s} = u' \frac{M_{ji}}{M} = k_{ji}$$

Actually, k_{ij} is the "residual variability" of the jth good for a "compensated variation" of the price p_i (Slutsky, 1915 (1952), 43)

Moreover, on the basis of the above results, Slutsky wonders about the empirical counterpart of the second derivatives of the utility function. He comes to a negative conclusion, which is of direct consequence on the distinction between complementary and competitive goods. Indeed, Slutsky provides the first explicit challenge to the Auspitz and Lieben criterion. His conclusion, at the end of the paper, is a final judgment:

"This whole edifice falls if one remains loyal to the formal definition of utility, for it is impossible to deduce from the facts of behavior the character (that is, the sign) of the second derivative of utility" (Slutsky, 1915 (1952), 54)

It is to be remarked that Slutsky's attitude toward this result contrasts strongly with Pareto's usual attempts at joining together different theoretical constructs.[53] Whereas Pareto aims at smoothing down the rough edges, Slutsky would not hesitate to dramatize the methodological consequences of the above result:

"The [ordinal] definition of utility (…) can serve as a basis for the entire theory of the budget. But since the values of the marginal utilities and their variations connected with variations in the quantities of goods remain undetermined, *obviously an irreconcilable conflict exists between the two aspects of the problem of utility*" (Slutsky, 1915 (1952), 54)

5.2 An exacerbation of Pareto's hesitations

Slutsky is very explicit about the role of utility in value theory. He adheres to the positivist idea of self-sufficiency of value theory but he does not want to renounce investigating relations between economic choice and psychical phenomena:

"If we wish to place economic science upon a solid basis, we must make it completely independent of psychological assumptions and philosophical hypotheses. On the other hand, since the fundamental concept of modern economics is that of utility, it does not seem opportune to disregard all connections existing between the visible and measurable facts of human conduct and the psychic phenomena by which they seem to be regulated. Utility must therefore be defined in such a way as to make it logically independent of every disputable hypothesis or concept, without however excluding the possibility of further research regarding the relations between the individual's conduct and his psychic life" (Slutsky, 1915 (1952), 27-28)

It must be clear that this attitude toward utility is of balancing (or counterbalancing) it with psychology. This perfectly echoes Pareto's remarks on the relations between economics and psychology. Pareto is calling for a provisional independent growth. The chapter II of the *Manual* (Introduction to social science) opens with the following statement:

"Clearly, psychology is fundamental to political economy and all the social sciences in general. Perhaps a day will come when the laws of social science can be deduced from the principles of psychology (…); but

[53] It is a pure speculation to wonder if Pareto had the slightest idea of this difficulty. From its discussion of the slope of indifference curves and indifference maps in the *Manual*, we cannot infer seriously a positive or negative answer. It is probable that Pareto derived general properties of the indifference maps (according to the complementarity relations at stake) on the basis of specific utility functions. Even in this case, he may have had to restrict the generality of the result in the plane.

> we are still very far from that state of affairs, and we must take a different approach" (Pareto, 1909 (1971), 29)

At first sight, Slutsky is on the road traced by Pareto. But the end of the paper will prove to be much less optimistic as to the referential status of psychology. Moreover, Slutsky underlines the intrinsic difference between ophelimity and the index function of utility (*i.e.* the ordinal index), and he pays attention not to confuse them. The first – ophelimity – is "a purely psychological concept" (Slutsky, 1915 (1952), 28) whereas the second one is a "happy construction, completely strict and abstract in all its aspects" (Slutsky, 1915 (1952), 28). From that, it is reasonable to infer that Slutsky is much less open to an obscure synthesis between the two concepts of utility: the psychological or hedonistic utility function should not be confused, intrinsically, with the formal or ordinal function.

5.3 Slutsky's silence: the mind-body problem

Slutsky's preliminary considerations are immersed in the never-ending question of the time in mathematical economics. How to build a mathematical theory of value independently of hedonistic (or psychological) foundations. Slutsky's own perception of the present state of economics is really that of a transitional state: the irreversible decline of the hedonistic foundations has not given place to a new, coherent, widely shared construction: "Thus, it is doubtful whether the hedonistic school today still preserves its predominance, although agreement among its opponents is far from universal" (Slutsky, 1915 (1952, 27)

The last sections (§12 and §13) of Slutsky's paper are devoted to the relations between the ordinal and the psychological utility functions. The main attitude of Slutsky is a testing attitude: to what extent is it possible to reconcile the two opposing concepts of utility?

From a methodological – even philosophical – point of view, Slutsky does not accept that economic behavior or economic choice could be independent of any psychological properties for the consumer. It would be misleading to interpret this definition as a stance in favor of hedonistic foundations of economics, and that may be a discreet but important shift from Pareto's position. Indeed, Slutsky never claims some priority for psychological facts on economic facts, and he stands on his desire to put both aspects (psychological and economical) on the same footing without ever mentioning any causal relationship between them. So, one way or the other, Slutsky looks for another definition of the formal-ordinal utility function that would allow for some connections with the hedonistic-psychological utility function. Otherwise stated, Slutsky's question is whether an appropriate amendment or twist of the formal-ordinal definition might be likely to induce some kind of structural relationships with the hedonistic one:

> "We therefore consider it necessary to complete the formal concept of utility in such a manner as to put the economic aspect of the problem of utility in close relation with the psychological one. Specifically, we propose to investigate whether the following definition is admissible.

> The utility of a combination of goods is a quantity which possesses the following properties: it is greater the more the combination is desired by the individual; and its variations are immediately perceptible by the subject" (Slutsky, 1915 (1952), 53)

What kind of structural relationships should be exhibited is not known, but it is right to note that Slutsky seems to be attached to the qualitative properties of the hedonistic function regarding the second derivative of the utility function. I mean that in the final analysis, he touches the fundamental question: if economics is doomed to entertain occasional relations with psychology, what kind of psychology will be most suitable?

So, what is at stake here is not the usefulness of the formal definition for economics, but instead the usefulness of the hedonistic definition for psychology. After all, it might be that the individual does not have full consciousness of the subjective motives of his action. To Slutsky's eyes, this is a question of "conviction" and "faith", and he will not exclude himself from such type of knowledge. As you may let yourself be persuaded, this critical point of Slutsky's paper arises precisely when complementarity enters the enters the scene.

> "If (…) we are convinced that the marginal utility of any one good decreases with an increase of its quantity; that moreover, for example, sugar and tea, salt and meat, etc., are complementary, while pork and mutton are normally competing, etc., it is obvious that this conviction can be founded only upon some sort of *internal evidence*, not on facts of economic conduct. The generality of this conviction authorizes us to call it *faith in the consciousness of economic conduct*. It is in fact common opinion that the motives by which we are guided, or at least factors parallel to these, manifest themselves, more or less clearly, in our consciousness, so as to enable us to perceive the increase and decrease in their intensity" (Slutsky, 1915 (1952), 55)

It is quite probable that Slutsky was not prepared for such radical conclusions. So, what about complementarity? The answer is very simple. According to Slutsky, complementarity can be nothing but a psychological subjective concept (what he calls "internal evidence") bearing consequences on individual consciousness, and hedonistic utility functions seem to be adapted to that aim. But within the economic world, there is no longer any reason why we should look for another definition in its place, be it an ordinal one. There is no need for it because there is no idea of it. To provide an ordinal definition would be at best a source of confusion, or worse it would argue for an instrumentalist attitude.

So, it is reasonable to infer from Slutsky's paper that not only did he provide the main structure of modern demand theory, and also that he was able to lay very relevant questions opening a new field of reflections on the relationships between psychology and economics, and even more on the type of psychological knowledge most suited for economics.

Concluding remarks

The concept of complementarity has been shown to be narrowly involved in the "ordinalist revolution", as well as in the development of the modern theory of demand. A few concluding remarks can help to state precisely why and to what extent complementarity is of central – and not marginal – importance for the history of ordinalism.

Firstly, it can be shown that the transformations of complementarity helped to disentangled demand theory from utility theory. Is there something intrinsic behind complementarity? Not necessarily. The utility function is proper to one person and it can enhance different properties with someone else (and some other utility function). Neither is it necessary to think that complementarity is an absolute relation. Two goods may well be complementary for a given basket and substitutes for another basket. Nevertheless, from the outset, utility was conceived as the analytical tool adapted to deal with complementarity. Of course, everything depends on the way utility is presented and used, but it is quite evident that all the authors who have the Auspitz-Lieben criterion in mind would consider seriously that the criterion imposes some restrictions either on demand behavior or on the price system. To be more precise, many authors would not have rebuked a purely observational and demand based presentation of substitutability. We have seen in Auspitz and Lieben and in Marshall many occurrences of this. Of course, it can be thought that this discussion was based, in their mind, to the utility-based definition. But it was also implicit that after all, the demand relation could have a sense in itself. As the Auspitz-Lieben criterion was challenged by preference-based definitions, the narrow link between utility and demand tended to loosen, as is very clear from Pareto onwards.

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Secondly, at least up to 1915, the transformations of the concept of complementarity do not seem to have been motivated by the search for an ordinal definition of complementarity. Even Johnson's ordinalist criterion appears to be quite incidental, although he questions the Auspitz-Lieben criterion. The motivations for a new definition were quite different and indirectly linked with ordinalism. Complementarity was viewed above all as a tool for improving demand analysis. This fact appears clearly in the writings of Pareto.

Thirdly, in the final analysis, a brief comment on Slutsky as a breaking point in the history of demand theory is in order. What is at stake in Slutsky's article is more than the analytical transition that will allow for the development of modern demand theory. It is also the psychological referent that economic theory puts in front of herself that has to be completely adapted to the methodological recommendations of the early ordinalists (Fisher and Pareto). Slutsky deserves much attention because he succeeds in isolating economic theory from a subjectivist conception of psychology, still bearing some marks from psychophysicians ideas (that of just perceptible increments, the rhetoric of a naturally ordered complex of sensations). For all that, do we have to consider Slutsky's views as encumbered with old-fashioned questions? On the contrary, as regards some investigations and

methodological attitudes such as that of Stigler (1950, (1965)) or Samuelson (1974), everything happens as if he had raised the fundamental questions of the microecomics of demand for the whole century.

Fourthly, I would like to anticipate a bit on the subsequent developments of this story. With the abandonment of the Auspitz-Lieben criterion, many problems will come on the way. What is the interest of a definition that would not be introspectively founded, or at least intuitively appealing? What kind of psychological referent will take the place of the old subjectivist and psycho-physiological referent? Once the utility relationships between two goods are no more significant, why continue to refer to a two goods case in order to represent complementarity? All this will be at stake in the later historical developments of the concept.

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