

# Strategic debt in a mixed duopoly

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May 2, 2007

## Abstract

This note studies the impact of the private firm's debt in a mixed duopoly. When the private firm has debt, the state-owned firm may decrease its output level so as to prevent the private firm from going bankrupt. The private firm uses strategically this effect of debt. The production of the state-owned firm is lower and the production of the private firm is higher than without debt. The social surplus is higher when the social cost of bankruptcy is low and lower if this cost is high.

Keywords : Mixed duopoly, strategic debt.

JEL Classification numbers : D43, L13, L32.

## 1 Introduction

The aim of this note is to analyse the impact of debt in a mixed duopoly, where a private firm is in competition in quantities with a state-owned firm. The objective of the private firm is to maximise its profit whereas the objective of the state-owned firm is to maximise the social surplus.<sup>1</sup> The strategic use of debt is an idea widely studied in economics literature. It has been shown that debt influences the rival firms (Brander and Lewis, 1986), suppliers or unions (Bronars and Deere, 1991) or regulation organization (Spiegel and Spulber, 1994). But this idea, in my knowledge, has not been studied yet in a mixed duopoly.

Brander and Lewis (1986) show that, in a private Cournot oligopolistic market with demand uncertainty, debt and limited liability induce firms to take more risky positions as suggested previously by Jensen and Meckling (1976). By increasing its debt level, a firm can move upwards its

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<sup>1</sup>There is no agency problem with the manager of the state-owned firm.

best reply function in the product competition stage thereby decreasing the equilibrium production level of its rival: debt has a strategic value. There is an important literature on this topic. In a two period model in which debt is repaid at the end of the last period, Glazer (1994) shows that debt is pro-competitive in the second period but allows some kind of tacit collusion in the first period because an increase in the rival's profit decreases its residual debt and makes it less aggressive in the last period. Considering a Bertrand competition model, Showalter (1995) derives that the strategic effect of debt depends on the type of uncertainty in the output market: if costs are uncertain, firms do not leverage, because more debt induces firms to decrease their prices, but, if demand conditions are uncertain, firms carry positive strategic debt levels in order to soften competition. In an entry framework, the incumbent wants to commit credibly to choose a low price in order to deter entry hence to be in debt if costs are uncertain and debt free if demand is uncertain (Showalter, 1999).

Debt can also have a negative strategic value. If there are bankruptcy costs, debt may induce firms to take less risk so to be less aggressive and their rivals to be more aggressive (Brander and Spencer, 1988; Parsons, 1997). Maksimovic (1988) shows that tacit collusion is more difficult to sustain when firms are more debt. In Bolton and Scharfstein (1990), debt decreases the probability of survival and therefore increases the probability that rivals will prey on it.

Debt can also be used in a strategic way in bargaining context. In Bronars and Deere (1991), shareholders and workers bargain on the sharing of firm's profit. So by issuing debt, shareholders reduce the profit of the firm and protect it from union appropriation. This effect can increase firm's investment by protecting it from workers appropriation (Dasgupta and Sengupta, 1993; Subramaniam, 1996). Perotti and Spier (1993) show how debt can be used to obtain wage concession from workers. If leverage is high, shareholders recapitalize the firm and undertake new investment only if workers give up some of their claims. Finally, debt can be used by regulated firm in order to influence bargaining with the regulator and obtain higher prices (Dasgupta and Nanda, 1993; Spiegel and Spulber, 1994; Spiegel, 1996).

We will see that debt can also be used strategically in a mixed duopoly to increase the private firm's profit. In a mixed duopoly, the state-owned firm produces a level of output such that the equilibrium price is equal to its marginal cost. So, generally, a state-owned firm produces more than a private firm would do. The quantities chosen by the firms being strategic substitutes, the quantity produced by the private firm is lower than the quantity it would produce if it were in competition with another private firm. So, in one hand, the total level of output is greater in a mixed duopoly than in a private duopoly; this first effect increases the social surplus. But, in the other hand, the

allocation of this output between the two firms is inefficient if the firms have decreasing returns to scale or if the cost of the state-owned firm is greater than the cost of the private firm; this second effect decreases the social surplus. The total effect of the ownership of firms on social surplus depends on the values of the parameters (De Fraja and Delbono, 1989).

In order to increase its profit, the private firm wishes to increase its production and the state-owned firm wishes to decrease its production to reduce the inefficiency of the production allocation. These commitments can be made by choosing a sequential timing (Pal, 1998; Jacques, 2004), by partially privatizing the state-owned firm (Matsumura, 1998), by committing on capacities (Nishimori and Ogawa, 2004; Lu and Poddar, 2005) or by manipulating the incentives of the firms' managers (Barros, 1995; White, 2001). The debt of the private firm is another way to commit on other equilibrium.

The private firm can modify the equilibrium by using debt strategically. Before to compete in quantities, the private firm can go in debt. If its debt is greater than its profit in the Cournot equilibrium without debt, it will go bankrupt if the state-owned firm does not change its level of output. But if this bankruptcy has social cost, the state-owned firm may accept to reduce its production in order to prevent the private firm from going bankrupt. The state-owned firm will decrease its production if the social cost of this output reduction is lower than the social cost of the bankruptcy. The private firm anticipates this behaviour of the state-owned firm and chooses the level of debt which maximises the output reduction of its competitor. So the debt of the private firm modifies the Cournot equilibrium of the quantity subgame. The debt has two effects, it decreases the global level of output and it reallocates the production between the firms more efficiently. So the effect of debt on the total surplus depends on the value of the parameters. Debt increases social surplus when the social cost of the bankruptcy is low and decreases it when this cost is high.

## 2 Model

There are two stages in the model. In the first one, the private firm negotiates a loan contract with outside investors. The financial market is perfectly competitive and the riskless interest rate is assumed to be zero. The amount borrowed is distributed between shareholders. In the second stage, the two firms choose simultaneously a quantity. Then the market is cleared. If the private firm is not able to reimburse its debt, it goes bankrupt. This bankruptcy has a social cost equal to  $B$ . Cost  $\lambda B$  is supported by the manager and the shareholders of the private firm and cost  $(1 - \lambda) B$  is supported by the stakeholders of the firm (suppliers, workers, consumers, etc). The results of the model do not

depend on the value of  $\lambda$ . The firms have constant marginal costs and the private firm (firm 2) is assumed to be more efficient than the state-owned one (firm 1):  $c_1 > c_2$ .<sup>2</sup> This assumption, that state-owned firms have higher costs than similar private firms, has empirical foundations. Megginson and Netter (2001) concludes (page 380) that: "*Research now supports the proposition that privately owned firms are more efficient and more profitable than otherwise-comparable state-owned firms*". Without loss of generality, we assume that  $c_1 = c$  and  $c_2 = 0$ . The inverse demand function is linear:  $p = \max(0, \alpha - \beta(q_1 + q_2))$ , where  $q_1$  and  $q_2$  are the quantities produced respectively by the state-owned and the private firms.

We note  $\pi_i$  the profit of firm  $i$ ,  $CS$  the consumers' surplus and  $W$  the social surplus.

### 3 Production game

We, first, present the production game equilibrium without debt. Then, we analyse how the introduction of strategic debt modifies this equilibrium.

#### 3.1 Without debt

As benchmark case, we compute the equilibrium of the second stage of the game when the private firm has no debt.

##### 3.1.1 Best reply function of the private firm

The objective function of the private firm is to maximize its profit:

$$\max_{q_2} \pi_2(q_2) = (\alpha - \beta q_1 - \beta q_2) q_2$$

So, its quantity best reply function is given by:

$$\frac{d\pi_2(q_2)}{dq_2} = 0 \Leftrightarrow q_2 = \frac{1}{2\beta} (\alpha - \beta q_1)$$

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<sup>2</sup>Assuming that the two firms have the same cost function with decreasing return to scale would give the same qualitative results.

### 3.1.2 Best reply function of the state-owned firm

The objective of the public firm is to maximize the social welfare, which is the sum of the firms' profits plus the consumers' surplus.

$$\max_{q_1} W(q_1, q_2) = \frac{1}{2}\beta(q_1 + q_2)^2 + (\alpha - \beta q_1 - \beta q_2 - c)q_1 + (\alpha - \beta q_1 - \beta q_2)q_2$$

So, its quantity best reply function is given by:

$$\frac{\partial W(q_1, q_2)}{\partial q_1} = 0 \Leftrightarrow q_1 = \frac{1}{\beta}(\alpha - \beta q_2 - c)$$

### 3.1.3 Equilibrium

From the quantity best reply functions of the firms, we deduce the Cournot equilibrium of the second stage of the game:

$$\begin{aligned} q_1 &= \frac{1}{\beta}(\alpha - 2c), & q_2 &= \frac{1}{\beta}c, & p &= c \\ \pi_1 &= 0, & \pi_2 &= \frac{1}{\beta}c^2, & CS &= \frac{1}{2\beta}(\alpha - c)^2 \quad \text{and} \quad W = \frac{1}{\beta}c^2 + \frac{1}{2\beta}(\alpha - c)^2 \end{aligned}$$

The equilibrium price is equal to the marginal cost of the state-owned firm and the private firm has a strictly positive market share only if it is more efficient than the state-owned firm. The level of output of the private firm depends only on the cost difference between the two firms; it is independent of the demand level.

## 3.2 With debt

Now we assume that the private firm has a debt of  $D$ .

Without uncertainty, the debt does not change the quantity best reply function of the private firm. But, it may modify the one of the state-owned firm. The public firm maximizes the social welfare, so it does care about the social cost of the private firm bankruptcy and it may modify its level of production in order to avoid the bankruptcy of the private firm.

We need to distinguish three cases according to the level of  $D$ .

If  $D \leq \frac{1}{\beta}c^2$ , the debt is lower than the Cournot profit of the private firm in the game without debt. The private firm can reimburse its debt even if the state-owned firm does not change its

production. So, the public firm has no reason to change its behaviour and the equilibrium is the same than without debt.

If  $D > \frac{1}{4\beta}\alpha^2$ , the debt is higher than the profit of a private monopoly. The private firm is not able to repay its debt even if the state-owned firm does not produce. So, the public firm cannot avoid the bankruptcy of the private firm and, as the social cost of bankruptcy does not depend on the sum reimbursed, the public firm has no reason to change its behaviour. So the equilibrium is the same than without debt, except that the social welfare is reduced by  $B$ .

If  $\frac{1}{\beta}c^2 < D \leq \frac{1}{4\beta}\alpha^2$ , the public firm can avoid the private firm bankruptcy if it reduces its production enough. So the public firm must compare the loss of consumers' surplus if it reduces its production and the social cost of bankruptcy if it does not.

If the state-owned firm does not change its level of output the private firm goes bankrupt. If the state-owned firm wants to prevent the private firm from going bankrupt, it must reduce its production. Then, its optimisation program becomes:

$$\begin{aligned} \max_{q_1} [\pi_1(q_1, q_2) + \pi_2(q_1, q_2) + S_c(q_1, q_2)] \\ \text{s.t. } \pi_2(q_1, q_2) \geq D \quad \text{and} \quad q_1 \geq 0 \end{aligned}$$

The constraint of this program is binding<sup>3</sup>,  $\pi_2(q_1, q_2) = D$ . The state-owned firm decreases its production just enough to the private firm can repay its debt and avoid bankruptcy. So, the quantity best reply function of the state-owned firm is:

$$q_1 = \frac{1}{\beta}\alpha - q_2 - \frac{1}{\beta} \frac{D}{q_2}$$

and the constrained solution of the Cournot game is:

$$\begin{aligned} \tilde{q}_1 &= \frac{1}{\beta}\alpha - 2\sqrt{\frac{D}{\beta}} \quad , \quad \tilde{q}_2 = \sqrt{\frac{D}{\beta}} \quad , \quad \tilde{p} = \beta\sqrt{\frac{D}{\beta}} \\ \tilde{\pi}_2 &= D \quad , \quad \tilde{\pi}_1 = -\frac{1}{\beta}\alpha c + (\alpha + 2c)\sqrt{\frac{D}{\beta}} - 2D \\ \widetilde{CS} &= \frac{1}{2\beta}\alpha^2 - \alpha\sqrt{\frac{D}{\beta}} + \frac{1}{2}D \quad , \quad \widetilde{W} = \frac{1}{2\beta}\alpha(\alpha - 2c) + 2c\sqrt{\frac{D}{\beta}} - \frac{1}{2}D \end{aligned}$$

This solution is equilibrium if and only if the state-owned firm does not wish change its production. If it increases its level of output on its best reply function without constraint, the private firm

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<sup>3</sup>The value of the private firm's profit is given before the debt is repaid.

goes bankrupt and we have:

$$\begin{aligned}
q_1^d &= \frac{1}{\beta}(\alpha - c) - \tilde{q}_2 = \frac{1}{\beta}(\alpha - c) - \sqrt{\frac{D}{\beta}} \quad , \quad p^d = c \\
\pi_1^d &= 0 \quad , \quad \pi_2^d = c\sqrt{\frac{D}{\beta}} \quad , \quad CS^d = \frac{1}{2\beta}(\alpha - c)^2 \\
\text{and } W^d &= c\sqrt{\frac{D}{\beta}} + \frac{1}{2\beta}(\alpha - c)^2 - B
\end{aligned}$$

The constrained solution is equilibrium if and only if  $\widetilde{W} \geq W^d$ . That is if and only if:

$$B \geq \frac{1}{2\beta}c^2 - c\sqrt{\frac{D}{\beta}} + \frac{1}{2}D$$

The constrained solution is equilibrium if the social cost of bankruptcy is high enough. If  $B$  is low, the output distortion to save the private firm is too high and the state-owned firm does not modify its production. So, for  $B < \frac{1}{2\beta}c^2 - c\sqrt{\frac{D}{\beta}} + \frac{1}{2}D$ , both firms behave as in the game without debt and the private firm goes bankrupt.

## 4 Debt contract and subgame perfect equilibrium

In stage 2, the gross profit of the private firm is just enough to repay its debt. So the shareholders of the private firm do not received dividend at the end of stage 2. But, at the end of stage 1, the amount borrowed is shared between the shareholders. This amount is the actual profit of the firm. This profit is an increasing function of the debt's level until the debt becomes too high and the state-owned firm chooses to not distort its output level to prevent private firm from bankruptcy. We have assumed that financial market is perfectly competitive. So outside investors accept to lend to the firm any amount for which the private firm will not go into bankrupt in stage 2. The amount to be repaid is equal to the amount borrowed because the risk less interest rate is equal to 0 and the debt of the firm is risk less. So the private firm chooses the higher level of debt for which the state-owned firm accepts to decreases its production.

$$D = \begin{cases} \frac{1}{\beta}c^2 + 2c\sqrt{\frac{2B}{\beta}} + 2B & \text{if } B \leq \frac{1}{8\beta}(\alpha - 2c)^2 \\ \frac{1}{4\beta}\alpha^2 & \text{if } B > \frac{1}{8\beta}(\alpha - 2c)^2 \end{cases}$$

The debt is equal to the amount borrowed, because, at the equilibrium, the probability of bankruptcy is nil. So, the subgame perfect equilibrium is:

For  $B \leq \frac{1}{8\beta} (\alpha - 2c)^2$  :

$$\begin{aligned}\tilde{q}_1 &= \frac{1}{\beta} (\alpha - 2c) - 2\sqrt{\frac{2B}{\beta}} \quad , \quad \tilde{q}_2 = \frac{1}{\beta} c + \sqrt{\frac{2B}{\beta}} \\ \tilde{p} &= c + \sqrt{2B\beta} \quad , \quad \tilde{\pi}_1 = (\alpha - 2c) \sqrt{\frac{2B}{\beta}} - 4B \quad , \quad \tilde{\pi}_2 = \frac{1}{\beta} c^2 + 2c\sqrt{\frac{2B}{\beta}} + 2B \\ \widetilde{CS} &= \frac{1}{2\beta} (\alpha - c)^2 - (\alpha - c) \sqrt{\frac{2B}{\beta}} + B \quad , \quad \widetilde{W} = \frac{1}{2\beta} (\alpha - c)^2 + \frac{1}{\beta} c^2 + c\sqrt{\frac{2B}{\beta}} - B\end{aligned}$$

For  $B > \frac{1}{8\beta} (\alpha - 2c)^2$  :

$$\tilde{q}_1 = 0 \quad , \quad \tilde{q}_2 = \frac{\alpha}{2\beta} \quad , \quad \tilde{p} = \frac{\alpha}{2} \quad , \quad \tilde{\pi}_1 = 0 \quad , \quad \tilde{\pi}_2 = \frac{\alpha^2}{4\beta} \quad , \quad \widetilde{CS} = \frac{\alpha^2}{8\beta} \quad , \quad \widetilde{W} = \frac{3\alpha^2}{8\beta}$$

The equilibrium price is an increasing function of  $B$ . The consumers' surplus is a decreasing function of  $B$ . When  $B$  increases, the output of the state-owned firm decreases and the output of the private firm increases, but by a smaller amount. So the total output decreases and that's why the equilibrium price increases. The profit of the state-owned firm is an increasing function of  $B$  for low values of  $B$  ( $B \leq \frac{1}{32\beta} (\alpha - 2c)^2$ ) and decreasing for high values of  $B$ . When  $B$  increases, the state-owned firm decreases its production but sales it at a higher price. For low values of  $B$ , the second effect dominates, whereas, for high values of  $B$ , it is the first effect which dominates. The profit of the private firm is an increasing function of  $B$ . The private firm increases its production and sales it at a higher price. The social surplus is an increasing function of  $B$  for low values of  $B$  ( $B \leq \frac{1}{2\beta} c^2$ ) and decreasing for high values of  $B$ . The strategic debt has two effects on the social surplus. First, it decreases the total output; this first effect reduces the social surplus. Second, it reallocates the production between the two firms and the private one is more efficient, so this reallocation reduces the cost of production and raises the social surplus. For low values of  $B$  ( $B \leq \frac{2}{\beta} c^2$ ), the second effect dominates and strategic debt increases the social surplus. For higher values of  $B$ , the first effect dominates and strategic debt decreases the social surplus.

We can note that, if the social bankruptcy cost is not too high ( $B \leq \frac{1}{8\beta} (\alpha - 2c)^2$ ),  $\tilde{q}_2$ ,  $\tilde{p}$  and  $\tilde{\pi}_2$  do not depend on the level of demand ( $\alpha$ ). So the results of the model do not change if we assume that the demand is uncertain when the private firm chooses its level of debt but known before the firms choose their levels of production (if the lowest level of demand is high enough).



## 5 Conclusion

This note has shown that the capital structure of a private firm in competition with a state-owned firm has a significant impact on the production strategies of the firms. When the private firm goes in debt, the state-owned firm reduces its production in order to avoid the private firm bankruptcy. So, by using debt strategically, the private firm can increase its production and its profits. The effect of strategic debt on social surplus can be positive or negative depending on the value of the bankruptcy cost.

This note is just a first step in the study of capital structure in mixed oligopoly. Other effects can appear if the demand is uncertain and the firms must choose their production before the demand is known. In this case, the production best-reply function of the private firm changes also, and not only the state-owned firm's one as in this note. The capital structure of the state-owned firm can also be relevant if we assume that this firm has a budget constraint. Debt may also have a significant impact on other strategic variables in a mixed duopoly as design of product, R&D, entry or technological choices. The examination of these issues is left for future research. In this note, we just try to have the simplest model as possible to illustrate one possible effect of capital structure in a mixed duopoly.

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