The conduct of Australian Monetary Policy and Evolution of Investment Expenditures

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Abstract :

Many economists agree that the optimal monetary rule is the one which, in face of technological shocks, succeed in maintain low (even zero) inflation rate. Nevertheless, the complexity of production process and irreversibility of investment decisions are not really considered. For all that, in a model designed to flesh out macroeconomics dynamics, it is important not to understate the investment behaviours of the variations of inflation. We investigate a non linear investment model for the australian economy to take into account the changes in monetary regime experienced since 1970 on capital accumulation process. This econometric analysis is supported by using Smooth Transition Regression models, in which the monetary orientations appear to play a major role in out of equilibrium dynamics of total capital stock. The empirical results suggest that interest rate policy seems to have more effect on transition periods than on equilibrium paths.

Key Words: Monetary Policy, Investment dynamics, Econometric modeling techniques, Non linear models.

JEL Classifications : C22, C52, E52, E58, O57.

I. Introduction

Many economists agree that the optimal monetary rule is the one which, in face of technological shocks, succeed in maintain low (even zero) inflation rate. The Central Banks have to remove interest rates to stabilize inflation pressures, this ultimate target is supposed to provide therefore stabilization of output gaps. This "theoritical consensus" about the aim of Central Bank is nevertheless criticable : on one hand, it supposes that the apparition of a new technology is sufficient to explain the rise of productivity. They do not consider time to build, or in other words, that returns on investment can not be immediate, because dependant of capacities of firms to integrate efficiently new capital processes. On the other hand, in the theoritical models designed to flesh out macroeconomic dynamics, the natural interest rate is often considered as unique and given by technlogy conditions. Following J.R. Hicks (1977), the equilibrium rate determination depends not only on real variables but also on monetary conditions that are the conditions required to take advantage of the new technology. The natural interest rate reflects just as well the alteration of technology as the conditions of reallocation of resources.

In a world with incomplete information and non instantaneous adjustments, it raises the question of the role of monetary policy and mainly the way by which the Central Bank can take place. With this paper, we try to understand this debate, usually called "rules versus discretion". We explore the role of australian monetary policy.

Australia has experienced several monetary regimes since 1970s. Following I.J. Macfarlane (1999), four main changes can be observed, not always smooth and widely dependent of Government orientations (until 1996). This singular situation provides an “empirical field” to appreciate the monetary regime the more appropriate to stimulate and to reinforce the capital accumulation process. So this paper is related to the discussion about the re-evaluation of the Reserve Bank of Australia (RBA) objectives as a monetary institution. Rather to focus on an optimal definition of monetary rules, it seems important to analyse the interactions between Central Banks policies and capital accumulation dynamics.

The plan of the paper is as follows : **Section II** presents the evolution of australian monetary regimes in historical perspective. **Section III** reports the econometric procedure used to estimate the relation between capital accumulation and monetary interventions. Within this section, we discuss about an important issue for monetary policy in face of technological shocks. **Section IV** offers concluding remarks.

Section II

Australian Monetary Regimes Changes

Since 70s, a moderated inflation rate was thought to be helpful in decreasing unemployment. The trade-off between growth and inflation supposed robust, Australian economy has experienced a high and persistent inflation rate in the mid-70s to the early 80s. The unemployment rate did not decline. On the contrary, it rose from 1.3 per cent in the early 70s to 6.5 per cent by 1978. M. Friedman (1969) pointed out that the Phillips Curve relates unemployment not to effective inflation but to unexpected inflation. Once inflation has become expected, the trade-off between inflation and employment disappears. Many economists agree that monetary policy has not been well managed, because of based on a pure discretion. Following Friedman's recommendations, "money matters" for macroeconomic fluctuations, with a long and variable lag. In view of the very imperfect predictability of these lags, discretionary policy can hardly do better than a fixed rule to expand the money supply.

By January 1976, the RBA moved to a monetary target for M3, influenced by monetarism theory. In fact, providing that the demand for money is stable, the inflation is linked to the growth of money supply. In that case, the control of M3 is sufficient to bring down inflation. Nevertheless, financial deregulation and economic structural changes (namely the exchange rate policy) caused an instability of the long-run relationship.

The RBA , in May 1985, adopted a new strategy, the so-called “Checklist”. This monetary orientation was built on a wide panel of economic variables. The main problem was to select the main indicators among the structural variables. The RBA gave importance, in turn, to stability in exchange rate, gross domestic product and prices. I.J. Macfarlane (1999) considers that “it failed to distinguish between the instrument of monetary policy, intermediate targets and ultimate targets”. This monetary orientation did not provide an appropriate regulation of economic system, the profile of inflation rate was enough erratic and persistent. Some economists have argued that the Checklist had contained the seeds of macroeconomic instability, mainly because Australia was still an inflation-prone economy. In defence, for the same period, Australia suffered financial shocks reinforced by the loss of confidence in Autralian macroeconomic policy. The Checklist policy faced too the implementation of flexible exchange rate system which intensified the instability of monetary system. At last, the australia government decided in 1987 to slow down the domestic demand to check the deterioration of external deficit by using interest rates. In 1988, Australia dipped into a deep recession, and at the same time, the RBA gave up the Checklist strategy.

The monetary policy adopted between 1988 and 1993 was purely discretionnary. The RBA was enough confronted to a high level of unemployment and a persistent inflation rate.

The critics were based on a lack of independence of the central bank (reinforced by the fact that the new Governor, Bernie Fraser, was a close associate of Treasurer Keating) and on the perverse effects of a dicretionnary policy (in particular, the critic of Kydland and Prescott about “time inconsistency”, 1977).

Moreover, there were two main oppositions about the determination of prices : first, the Monetarist Theory supposed, as explained above, that the prices were endogeneous to the evolution of the money supply growth. Second, the Government thought that inflation rate could be monitored by a control of wages variations. It is probably the reason why the monetary policy was assigned to other objectives than an pure inflation targeting.

The depression helped to obtain a lower inflation rate in 1992. The RBA adopted, in 1993, a model of inflation targeting. Nethertheless, it seems that this monetary policy rule is quite different from the one described by R. Clarida, J. Gali, M. Gertler (2000) or by M. Woodford (2003). By fact a policy based on an inflation targeting is sufficient to support a stable growth. There is no place to temporary monetary adjustments according to real misadjustments. « Obtaining a more desirable pattern of responses to random disturbances therefore requires commitment to a systematic policy rule and not just a (one time) adjustment of the bank’s targets » (Woodford 2001 p. 26).

Or, following the Governor I.J. Macfarlane (1999), “there was a tendency to suggest that the single objective of low inflation meant that central bank should pay no attention at all to other economic variables...if primacy is given to maintaining low inflation, the effects on output and employment had to be taken into account and had to influence monetary policy actions”.

To recognize real effects to monetary policy implies a priori to take into account the main characteristics of innovation processus. J. Schumpeter (1934) supports the idea that to implement new production processes, it requires additionnal financial resources. By definition, a technological shock provides a break in the current steady state. The firms have to reorganize the whole production process to integrate and to run the new technology. And during this transition period, the wages are paid without immediate compensation. Obviously, if there is not instantaneous return of investment, the costs of production will arise, generating an inflation rate more important, at least temporarily.

A typical dylemn for monetary institution emerges in face of technological shocks : the Central Bank can try to fight immediatly the inflation gap, exarcerbeting by fact the unemployment and the gap between real and potential growth. Alternatively, monetary autorities can be more flexible to facilitate the integration of new processes by decreasing liquidity constraints of firms. So, in this perspective, the monetary policies have to be modulated according to the source and the nature of instability.

Section III

The Investment Transition Function :

A Smooth Transition Regression Model

We begin by implementing S. Tevlin and K. Whelan (2000)'s linear model. This basic model considers the adjustment costs of the capital stock between two periods, and also the deviations separating the effective stock from the optimal one. Following Tevlin S. and Whelan K.[†] (2000), the capital accumulation process is defined by the following equation:

$$k_t = \lambda k_{t-1} + (1 - \lambda)(1 - \theta\lambda)(\kappa(L)y_t - \sigma\,\mu(L)r_t) + \eta'_t \quad (1)$$

with $\kappa(L)$ and $\mu(L)$, respectively the approximations of the effects of the product and the capital cost on k_t^{**}.

If we suppose that the technological bias cannot be observed, the estimated equation becomes:

$$K_t = \alpha + \gamma K_{t-1} + \sum_{i=0}^{N} \beta_i Y_{t-i} + \sum_{i=0}^{N} \gamma_i R_{t-i} + u_t \quad (2)$$

with K_t : the capital stock ;

Y_t : the GDP ;

and R_t : the capital cost (i.e. Hall-Jorgenson Formula).

[†] For more details on their modeling, see S. Tevlin et K. Whelan, 2000, pp. 11-12. See Appendice 1 for a precise description of the equations.

The basic linear model we use is slightly different from Tevlin S. and Whelan K. (2000), at least for the following reasons:

1. We introduce a liquidity constraint through the Cash Flow/Capital Stock ratio, and define it as an exogenous variable. Indeed, and according to J. Hicks (1973), the liquidity constraint influences capital accumulation processes through its impact on firms' self-financing capacities. We thus consider this issue as a relevant one.

2. Because we are interested in studying the role of the Central Bank in the capital accumulation process, we took the short term real interest rate as the relevant variable[‡]. The short term real interest rate is then calculated as the difference between the short term nominal interest rate and the expected inflation (i.e. measured by the average inflation rate over the last three quarters).

The estimated equation then takes the following form:

$$K_t = \alpha + \lambda K_{t-1} + \sum_{i=0}^{T} \beta_i Y_{t-i} + \sum_{i=0}^{T} \gamma_i IR_{t-i} + \sum_{i=0}^{T} \phi_i CF_{t-i} + \psi_t \qquad (3)$$

[‡] We do not use the commonly used Hall-Jorgenson rental rate formula. For a presentation of the Hall-Jorgenson formula, see Tevlin S. and Whelan K. (2000, Appendices, pp.22-23).

with, K_t : the total physical capital stock ; $Y_{t,}$ the GNP ; IR_t : the short term real interest rate and CF, the liquidity constraint.

The data are quarterly time series spanning for the period 1971:01 to 2005:04. We obtain the data from RBA statistics. We use as short term interest rate the average cash rate in each quarter expressed in monthly rate. The baseline inflation measure is the CPI measure.

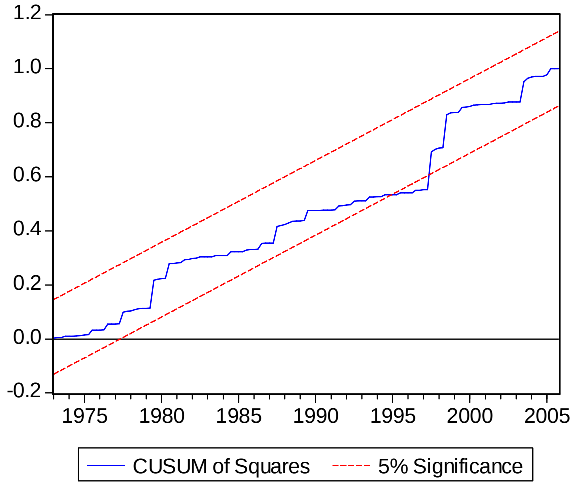
We use quaterly data of GDP at constant prices and Cash-Flow detrended. This linear model is estimated on first difference (the test of Augmented Dickey-Fuller reveal an unit root for each serie). Table I reports OLS estimates of the linear model.

Table I : Linear Model Estimates (1971:01 – 1995:04)

γ	β	Γ	Φ
0.804** (0.056)	0.155* (0.074)	-0.008* (0.0033)	0.76* (0.32)
R^2 : 0.629	Adjusted R^2: 0.621	Breusch Godfrey Prob : 0.3	AIC : -6.449

Standard Errors are reported in parentheses. ** : significant at 1% level. *: significant at 5% level.

Graph 1 : Test Cusum Squares



Graph I reports the result of Cusum Squares Tests over the period. A simple look a the graph is then sufficient to conclude to an instability of the coefficients. The structural breaks can occur when an important exogeneous variable or lags of an existing exogenous variable are omitted. We explore the robutness of our results along a number of dimensions, by testing different structures of lags. The instability of parameters remains whatever the alternative horizons for each variable. According to the historical perspective presented above, Australia faced main structural and institutional changes over the whole period. Certainly, the investment behaviours became instable because of the evoluting policy and growth conditions.

How can we explain this econometric results ?

We know that the way by which a monetary policy should take place, depends on the nature of economic instability. If the last results only of sticky prices, then the ability of supply and demand shocks to generate cycles depends only on the nature of the monetary policy rule. Providing that Central Banks adopt a proactive stance toward controlling inflation, the output gap and mainly the change in accumulation of capital remain stable. The real interest rates should affect in a constant manner the investment decisions. In our framework, it is not the case. Two main reasons could be advanced : in one hand, RBA have experienced four monetary regimes, and it is only in 1993 that RBA have adopted a model of inflation targeting. But in that case, we should obtain a stability of the coefficients since 1993-1994. The tests of structural stability accross the sub period (1994:01-2005:04) reveal differences in the conduct of monetary policy on investment decisions. On the other hand, real perturbations can be explained by the charasteristics of innovation process. To give up the hypothesis of instantaneous returns of investment implies to focus on the constraints that raise in transition period that is at the time of rupture of steady state.

The relation between monetary and real phenomenoms are perhaps more complex than supposed by current assertion. Following I.J. Macfarlane (1999), if monetary regimes are modulated by the nature of economic disturbances, the relation between capital accumulation processes and monetary policy can not be surely linear as supposed by S. Tevlin and K. Whelan's model (2000).

We next explore the use of a non linear model, called Smooth Transition Regression to take account of structural changes in capital accumulation process. We implement a *Smooth Transition Regression* Model (STR model) for Australia so as to introduce the changes in investment behaviours during the transition phases. STR models are indeed grounded on *transition functions* that allow to such structural changes.

Following Teräsvirta T. (1994) and Van Dijk D. and *al.* (2000), STR models take the following form:

$$Y_t = \left[A + B \times G\left(s_t ; \gamma ; c\right) \right] + \varepsilon_t \qquad (4)$$

with, y_t : the dependant (endogenous) variable; A and B : the vectors of parameters; G (s_t ; γ ; c) : a continuous function taking its values between 0 and 1 ; γ : the speed of changes of the constitutive values of the transition function (i.e. if γ is high then transition is fast from one regime to another) ; ε_t : a white noise ; c : the threshold (for regimes to change) ; s_t : the lagged exogenous variables, or the exogenous variable, or else a function of lagged exogenous variables.

There exists different transition functions, each of them modeling a specific adjustment mechanism of the endogenous variable (i.e. variable's growth regimes, D. van Dijk et *al.* 2000). A commonly used function is the following:

$$G (s_t ; \gamma ; c) = (1 + \exp\{(-\gamma(s_t - c)\})^{-1}, \quad \gamma > 0, (5)$$

Equation (5) is a logistic function that admits monotonic dynamics depending on the values of s_t. The corresponding STR model is called logistic STR (LSTR1). Van Dijk D. and *al.* (2000) show that LSTR1 models could describe potentially different smooth transitions between phases of expansion and phases of contraction of the endogenous variable.

The logistic function is not the only one for modeling transition dynamics. If we adopt the following transition function:

$$G (s_t ; \gamma ; c) = (1 + \exp\{(-\gamma(s_t - c)^2\}, \quad \gamma > 0 (6),$$

We obtain smooth adjustment dynamics that are similar between phases of expansion and phases of contraction of the endogenous variable. The STR model that incorporates this transition function is called exponential STR (ESTR).

It is also possible to use the second order logistic function:

$G(s_t ; \gamma ; c) = (1 + \exp\{(-\gamma(s_t - c_1)(s_t - c_2)\})^{-1}, \quad c_2 \geq c_1$ (7)

Using this function leads to the definition of a STR model called LSTR2.

To select a relevant STR model, we apply the methodology introduced by Teräsvirta T. (1994), that is a serie of embedded (overlapping) tests. From this methodology, we conclude that STR models are relevant to describe capital accumulation dynamics, and more particularly LSTR1 models (equation (5)).

The generic model we implement is thus the following:

$$K_t = \alpha + \lambda K_{t-1} + \sum_{i=0}^{T} \beta_i Y_{t-i} + \sum_{i=0}^{T} \gamma_i IR_{t-i} + \sum_{i=0}^{T} \phi_i CF_{t-i} \quad \text{(Linear Part)}$$

$$+ \left(\sum_{i=0}^{T} \varphi_i Y_{t-i} + \sum_{i=0}^{T} \mu_i IR_{t-i} + \sum_{i=0}^{T} \omega_i CF_{t-i}\right) \times \left(1 + e^{\{-\tau \times (K_{t-1} - \psi)\}}\right)^{-1} \text{(Non Linear Part)}$$

With :

λ, β_i, γ_i, ϕ_i : the coefficients of the linear model ;

φ_i, μ_i, ω_i : the potential parameters of the function ;

τ , the speed of the change of the constitutive values of the transition function ;

ψ, the regimes' change threshold ;

the variables are identical to those of the basic model.

At this stage, we have to specify the parametric structure of the model. Estimation of the parameters of the transition function (i.e. the vector of the initial values of the non linear component) is grounded on the results of the linear model (Non Linear Least Squares).

We apply Student Tests to select the relevant variables involved in the adjustment process. We therefore eliminate the variables that are non significant at a 5% threshold (precision level). One should notice that the transition functions essentially reflect the *changes of the speed* of the capital accumulation process. Comparing the transition functions does not offer any insight about changes of growth regimes, but only about dynamic reactions of the accumulation processes. Indeed, the linear restriction tests do not conclude to the rejection of the *non-null* hypothesis of the constant parameter of the transition function.

According to Van Dijk D. et *al.* (2000), if the endogenous variable is a growth rate, and if the constant parameter is *null*, then STR models are able to discriminate periods of negative growth (depression) from periods of positive growth (expansion). This is not the case here. Within STR models, the speed of adjustment of the capital stock is captured by the shape of the transition functions. By definition, those functions are defined over [0 ; 1].

We thus have the following relations:

1. The speed of adjustment increases when the transition function tends to 1.
2. Conversely, the speed of adjustement decreases when the transition function tends to 0.

One should note that the shape of the transition functions shows the reactions of the process to variations of the exogenous variables and shocks.

The model estimated is the following:

$$K_t = \alpha + C(1) \times K_{t-1} + C(2) \times Y_t + C(3) \times IR_{t-2} + C(4) \times \mathrm{CF}_{\text{t-1}} \quad \text{(Linear Part)}$$
$$+ \ (\mathrm{C}(5) \times IR_{t-3}) \times \left[1 + e^{-C(6)(K_{t-1} - C(7))} \right]^{-1} \ \text{(Non Linear Part)}$$

With:

C(1) to C(4): the coefficients of the linear model

C(5): the effective parameter of the transition function

C(6): the speed of change of the constitutive values of the transition function

C(7 : the regime's change threshold.

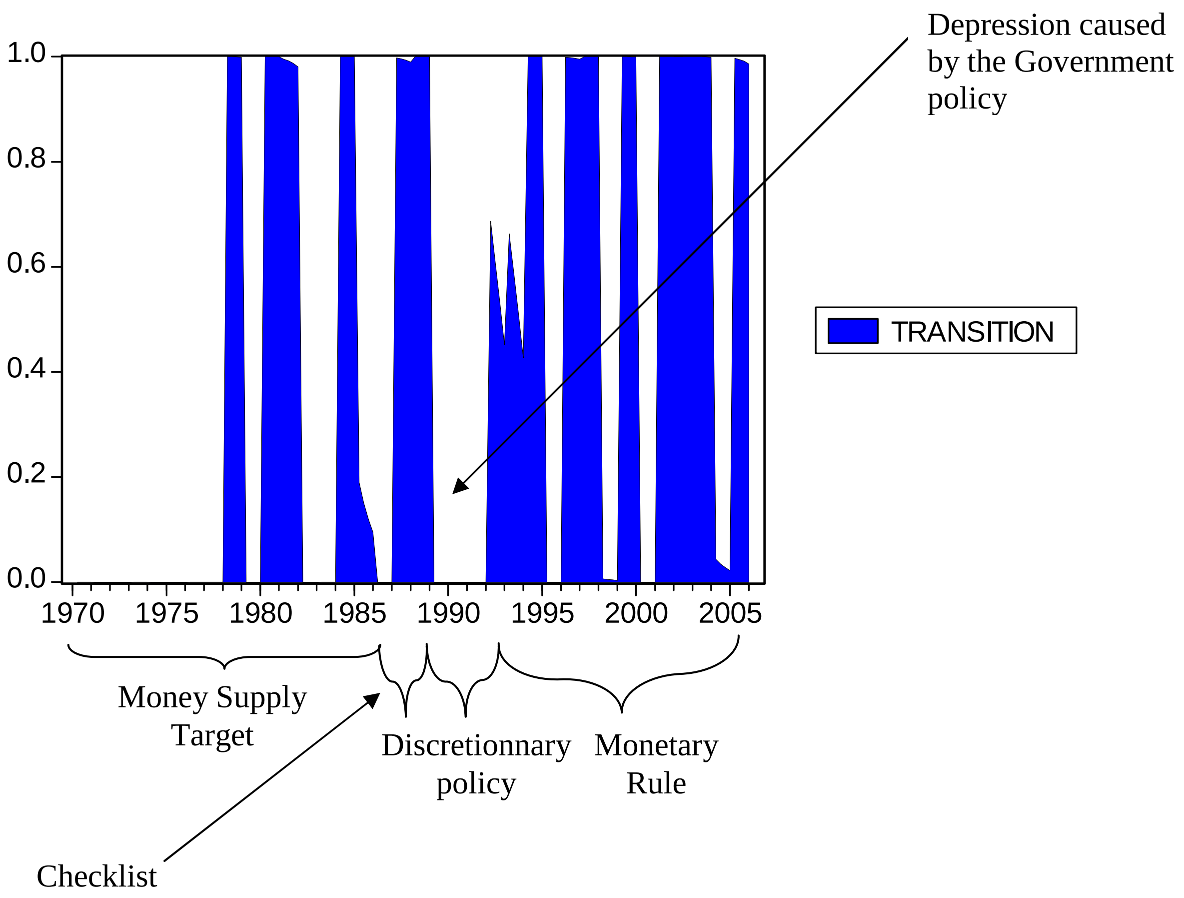
Table 2 reports the estimates of the Australian STR Model in first difference using Non Linear Least Squares. In fact, the signs and the magnitude of the estimated parameters remain unchanged for the linear component. The graph 2 reports the cyclical behaviour of investment decisions. Note that specific transition phases are associated with different monetary regimes. And note that only real interest rate is associated with this transition function.

Table 2:

C(1)	C(2)	C(3)	C(4)	C(5)	C(6)	C(7)
0.835** (0.057)	0.147* (0.070)	-0.007* (0.0031)	0.783* (0.31)	-0.043* (0.018)	500.11 (934)	0.0135** (0.004)
R^2 : 0.666		Adjusted R^2: 0.648		Breusch-Godfrey Prob. : 0.2		AIC : -6.524

Standard Errors are reported in parentheses. ** : significant at 1% level. *: significant at 5% level.

Graph 2 : Transition Function



From Graph 2, we observe a diversity of the shapes of the transition dynamics (i.e. investments' volatility is different from one monetary regime to another). The speed of adjustment of the capital stock and its (time-lagged) reactions to exogenous shocks (in particular in 1990) are thus different and dependent of the monetary policy. It is linked to the fact that the real interest rate is the only exogenous variable associated to the transtion function. So, the variations of the interest rate are directly involved in the observed changes of the speed of capital accumulation. This means implicitely that the monetary interventions have generated some distorsions over the reactions of investment expenditures. Such distorsions are captured by a strong variability of the transition function, whatever the monetary regime.

Therefore, and because real interest rate is closely linked to deceleration or acceleration of capital accumulation processes, the articulation between real and monetary variables are much more complex than the usual assertions suppose. A stabilization of prices does not warrant a stabilization of growth, or in our case, a stabilization of investment decisions. On the contrary, following Austrian Theory, in particular J.R. Hicks (1973), during the transition phases in which firms have to support additionnal liquidity constraints, the prices are increasing, stemming from a relative increase of capital goods prices. In turn, the decline in inflation, due to a pure inflation target, appears to generate a decline in the investment decisions by increasing the liquidity constraints of firms.

Under this type of monetary regime, we should observe a strong variability of investment linked to a revision of economic forecasting. In other words, monetary policy in place is in itself a source of macroeconomic instability. It could explained the reason why we can observe an important variability in the values of the transition function associated to the short term interest rate since 1995.

We note also that even if the monetary rule adopted in 1995 do not produce a stabilization of investment expenditures, it remains the more appropriate monetary regime compared with the other monetary policies. The monetary target respectevely on M3 and on relative prices stability have generated opposite reactions of capital accumulation. Indeed, it seems that the periods of increasing investment are more important since 1995 than for the period between 1976-1985. The climate of confidence, the evolution of external balance, the domestic consumption, and so on, have not to be neglected to explain the investment difference between those two sub-periods.

IV. Conclusion

This econometric study has emphasided how differences in monetary regimes might account for differences in investment behaviors since 1970. It reveals mainly that a pure discretionnary policy or a pure inflation target leave open the possibility of an economic instability. This feature can be explained by the temporal dimension of production process. We know that the definition of an optimal monetary policy is linked to the source and the nature of macroeconomic instability. The distorsions between supply and demand are not sufficient a priori to define an optimal monetary rule, because they do not consider the specific pro-cyclical character of the investment decisions.

The transition periods, and in particular, the monetary conditions of investment expenditures, represent a decisive variable in the conduct of monetary policy that we should not neglected. To stabilize the reactions of economic decision-makers is then a serious task since it could greatly simplify the coordinating and regulatory role of the Reserve Bank of Australia by improving convergence and reducing structural disparities.

As D.H. Robertson has emphasized (1926, p.39), « the aim of monetary policy should surely be not to prevent all fluctuations, but to permit those which are necessary to the establishment of appropriate alterations in output and to repress those which tend to carry out the alterations in output beyond the appropriate point ». In this respect, and because of its crucial position, the RBA is confronted to serious challenges, and the stabilization of investment decision-makers' expectations is part of these challenges. Improving agents' confidence for the RBA's monetary orientations, is then a decisive aspect of the political debate about the objectives, the means and the efficiency of monetary policies. The expected effects could be a reduction of the range of investment decisions' volatility, and a validation of the major RBA's objectives.

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Appendice : The linear Model

Formally, Tevlin S. and Whelan K. (2000)'s model consists in minimizing the following quadratic function :

$$\text{Min } E_t \sum_{m=0}^{\infty} \theta^m \left([k_{t+m} - k^*_{t+m}]^2 + \alpha [k_{t+m} - k_{t+m-1}]^2 \right) \quad (1)$$

with θ : the actualization rate.

The first order conditions of this program are :

$$E_t \left[-k_{t+1} + \left(1 + \frac{1}{\theta} + \frac{1}{\alpha\theta} \right) k_t - \frac{1}{\theta} k_{t-1} - \frac{1}{\alpha\theta} k^*_t \right] = 0 \quad (2)$$

To compute the equation (1), we consider the following caracteristic equation:

$$X^2 - \left(1 + \frac{1}{\theta} + \frac{1}{\alpha\theta} \right) X + \frac{1}{\theta} = 0 \quad (3)$$

Equation (3) admits two (unit) roots that are defined by, $0 < \lambda < 1$, et $\frac{1}{\lambda\theta}$.

If we consider $F\,K_t = K_{t+1}$ et $L\,K_t = K_{t-1}$, equation (2) takes the following form:

$$-F\,K_t + \left(1 + \frac{1}{\theta} + \frac{1}{\alpha\theta} \right) K_t - \frac{1}{\theta} L K_t = \left(-F + 1 + \frac{1}{\theta} + \frac{1}{\alpha\theta} - \frac{1}{\theta} L \right) K_t \quad (4)$$

Multiplying equation (4) by FL, we obtain (5):

$$\left(-F^2+\left(1+\frac{1}{\theta}+\frac{1}{\alpha\theta}\right)F-\frac{1}{\theta}F\right)LK_t=-(F-\lambda)\left(F-\frac{1}{\lambda\theta}\right)LK_t$$

The first order conditions are:

$$E_t\left[-(F-\lambda)\left(F-\frac{1}{\lambda\theta}\right)LK_t-\frac{1}{\alpha\theta}K_t^*\right]=0 \quad (6)$$

$$-K_{t+1}+\lambda K_t+\frac{K_t}{\vartheta\lambda}-\frac{1}{\vartheta}K_{t-1}-\frac{1}{\alpha\theta}K_t^*=0 \quad (7)$$

Multiplying equation (7) by θλ, we obtain (8):

$$-\theta\lambda K_{t+1}+\lambda^2\theta K_t+K_t-\lambda K_{t-1}-\frac{\lambda}{\alpha}K_t^*=0$$

Substituting and considering:

$0 < \lambda < 1$ and $0 < \theta < 1$ so that limit $(\lambda\theta)^n = 0$ when $n \to +\infty$,

we obtain :

$(\lambda\theta)^n (I - \lambda F) K_{t+n} = 0$ (9)

A solution of the maximization program is then:

$$k_t = \lambda k_{t-1} + \frac{\lambda}{\alpha}E_t\left[\sum_{n=0}^{\infty}(\theta\lambda)^n k_{t+n}^*\right] \quad (10)$$

Taking equations (5) et (6), the coefficients should respect the followings:

$- (1 - \lambda)*(1 - 1/\lambda\theta) = 1/\alpha\theta$

$-1 + \lambda + 1/\lambda\theta - 1/\theta = 1/\alpha\theta$

Multiplying by $\theta\lambda$, we obtain the following conditions:

$-\lambda\theta + \lambda^2\theta + 1 - \lambda = \lambda/\alpha$

$(1 - \lambda)(1 - \lambda\theta) = \lambda/\alpha$ (11)

Taking conditions (11), equation (10) takes the form:

$\Delta k_t = (1 - \lambda)(k_t^{**} - k_{t-1})$ (12)

with $k_t^{**} = (1 - \lambda\theta) E_t\left[\sum_{n=0}^{\infty}(\theta\lambda)^n k_{t+n}^{*}\right]$.

If we compute profit maximization, we obtain the following first order conditions (one can note that S. Tevlin et K. Whelan (2000) use a CES production function):

$k_t^* = \eta_t + y_t - \sigma r_t$

with η_t, corresponding to the effects of technologic changes ; y_t and r_t the amounts of sales and the short term real interest rate.