

Technical Appendix to  
**The Impact of Dependence among Voters' Preferences with Partial Indifference<sup>1</sup>**

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1) Condorcet Efficiency of PR (with  $\alpha = k/n$ )

$$CE_{CW}^{\infty}(PR, \alpha, FIAC) = [1/57153600 (544416582887649 \alpha^{11} - 1233547576052400 \alpha^{10} + 1381394796264000 \alpha^9 - 2354178724632000 \alpha^8 + 4104524838960000 \alpha^7 - 5494304068344000 \alpha^6 + 5501396660760000 \alpha^5 - 4002860357496000 \alpha^4 + 2034941856948000 \alpha^3 - 684454018248000 \alpha^2 + 137278890548800 \alpha - 12479899140800) / (4032969 \alpha^{11} - 10344060 \alpha^{10} + 20040630 \alpha^9 - 49394240 \alpha^8 + 99197280 \alpha^7 - 139767264 \alpha^6 + 140492352 \alpha^5 - 101312640 \alpha^4 + 51222600 \alpha^3 - 17222920 \alpha^2 + 3459456 \alpha - 314496)],$$

for  $0 \leq \alpha \leq 1/12$ ;

$$= [-1/11999512627200 (112181866735991926752 \alpha^{16} - 344969465036432140800 \alpha^{15} + 276181442600829696000 \alpha^{14} + 200675176033764096000 \alpha^{13} - 762360969242396160000 \alpha^{12} + 1129685992617695385600 \alpha^{11} - 1150777453002931353600 \alpha^{10} + 839830064482176768000 \alpha^9 - 427179101894633088000 \alpha^8 + 143697468569630976000 \alpha^7 - 28821655417537382400 \alpha^6 + 2620163807006054400 \alpha^5 + 601001856000 \alpha^4 - 16595712000 \alpha^3 + 317520000 \alpha^2 - 3763200 \alpha + 20825) / (\alpha^5 (4032969 \alpha^{11} - 10344060 \alpha^{10} + 20040630 \alpha^9 - 49394240 \alpha^8 + 99197280 \alpha^7 - 139767264 \alpha^6 + 140492352 \alpha^5 - 101312640 \alpha^4 + 51222600 \alpha^3 - 17222920 \alpha^2 + 3459456 \alpha - 314496))],$$

for  $1/12 \leq \alpha \leq 1/9$ ;

$$= [-1/23999025254400 (258412979442136431879 \alpha^{16} - 798896517177352532400 \alpha^{15} + 683515536345950805000 \alpha^{14} + 312420634197609798000 \alpha^{13} - 1485052309434975781500 \alpha^{12} + 2246778452203702981200 \alpha^{11} - 2298579100793189814600 \alpha^{10} + 1679122627201801782000 \alpha^9 - 854282872102847937750 \alpha^8 + 287386692630371334000$$

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<sup>1</sup> The authors thank Halim El Ouafdi for his contribution to the derivation of the final representations given in this appendix.

$\alpha^7-57642607035535321800 \alpha^6+5240281167981896400 \alpha^5+3527815882500 \alpha^4-$   
 $118707498000 \alpha^3+2814021000 \alpha^2-41924400 \alpha+295225)/(\alpha^5 (4032969 \alpha^{11}-10344060$   
 $\alpha^{10}+20040630 \alpha^9-49394240 \alpha^8+99197280 \alpha^7-139767264 \alpha^6+140492352 \alpha^5-101312640$   
 $\alpha^4+51222600 \alpha^3-17222920 \alpha^2+3459456 \alpha-314496))],$

for  $1/9 \leq \alpha \leq 1/6$ ;

$= [-1/23999025254400 (6822942035573718279 \alpha^{16}-52264954401936519600$   
 $\alpha^{15}+46116784965249621000 \alpha^{14}+238422999689203014000 \alpha^{13}-921443041016974885500$   
 $\alpha^{12}+1708965544039454610000 \alpha^{11}-2000739866290754589000$   
 $\alpha^{10}+1565451353709513270000 \alpha^9-822475164561830529750 \alpha^8+280681077084750342000$   
 $\alpha^7-56567107113155238600 \alpha^6+5109387665428846800 \alpha^5+11915942262178500 \alpha^4-$   
 $786670106922000 \alpha^3+35630875917000 \alpha^2-991164022800 \alpha+12776465800)/(\alpha^5 (4032969$   
 $\alpha^{11}-10344060 \alpha^{10}+20040630 \alpha^9-49394240 \alpha^8+99197280 \alpha^7-139767264 \alpha^6+140492352$   
 $\alpha^5-101312640 \alpha^4+51222600 \alpha^3-17222920 \alpha^2+3459456 \alpha-314496))],$

for  $1/6 \leq \alpha \leq 2/9$ ;

$= [1/647973681868800 (1512613388213406632259 \alpha^{16}-7101420028988147147376$   
 $\alpha^{15}+17074787179931425483560 \alpha^{14}-29721243845104668012240$   
 $\alpha^{13}+44790043068406492414740 \alpha^{12}-58411972429767014084208$   
 $\alpha^{11}+59691316587055645676184 \alpha^{10}-44281101194499780570960$   
 $\alpha^9+22763680400483324191890 \alpha^8-7698927007913853151440$   
 $\alpha^7+1547702207987774903064 \alpha^6-140623804872446483952 \alpha^5-56070962804607660$   
 $\alpha^4+1816814476920240 \alpha^3+22814263037160 \alpha^2-4194950823504 \alpha+109647645352)/(\alpha^5$   
 $(4032969 \alpha^{11}-10344060 \alpha^{10}+20040630 \alpha^9-49394240 \alpha^8+99197280 \alpha^7-139767264$   
 $\alpha^6+140492352 \alpha^5-101312640 \alpha^4+51222600 \alpha^3-17222920 \alpha^2+3459456 \alpha-314496))],$

for  $2/9 \leq \alpha \leq 1/4$ ;

$= [1/2249908617600 (57825105997174502003334 \alpha^{16}-279302955615974756795616$   
 $\alpha^{15}+625769034189740155164240 \alpha^{14}-869890031926957267017120$   
 $\alpha^{13}+853802169833958812649000 \alpha^{12}-643627729846395385928928$   
 $\alpha^{11}+394338991646298716153136 \alpha^{10}-199517539559122064760480$   
 $\alpha^9+80543738305241245534500 \alpha^8-24075398681843175707040$   
 $\alpha^7+4779445323766516459056 \alpha^6-534633035349879569376 \alpha^5+28878297797253177000$   
 $\alpha^4-2435551700973824160 \alpha^3+142427045555378640 \alpha^2-5161670595742752$   
 $\alpha+87365990111225)/(49567630400 \alpha^{16}-288352278272 \alpha^{15}+752425261440 \alpha^{14}-$

$1181716910080 \alpha^{13} + 1263251691520 \alpha^{12} - 986236532736 \alpha^{11} + 588941889536 \alpha^{10} - 276340613120 \alpha^9 + 102040773120 \alpha^8 - 28745699840 \alpha^7 + 5797535744 \alpha^6 - 774638592 \alpha^5 + 70120960 \alpha^4 - 6072640 \alpha^3 + 363840 \alpha^2 - 13484 \alpha + 233$ ],

for  $1/4 \leq \alpha \leq 1/3$ ;

$= [-1/1124954308800 (6178949986418658900684 \alpha^{16} - 14845117089098043570816 \alpha^{15} + 2634056988223226358240 \alpha^{14} + 10722158655572263898880 \alpha^{13} + 41473656868739421690000 \alpha^{12} - 150761970432461651372928 \alpha^{11} + 216196082508021121842336 \alpha^{10} - 183979258677882592404480 \alpha^9 + 103009936858009186005000 \alpha^8 - 39148707551888853959040 \alpha^7 + 10139866095548611277856 \alpha^6 - 1820949214120274923776 \alpha^5 + 254002658549524794000 \alpha^4 - 32471505377207084160 \alpha^3 + 3205551348848348640 \alpha^2 - 221273690464153152 \alpha + 7747963135368575) / (34783088231 \alpha^{16} - 214666362320 \alpha^{15} + 598699243320 \alpha^{14} - 987322942480 \alpha^{13} + 1051253088340 \alpha^{12} - 728786267664 \alpha^{11} + 296034530792 \alpha^{10} - 24888852560 \alpha^9 - 50147298630 \alpha^8 + 35499452560 \alpha^7 - 13223826616 \alpha^6 + 3314425296 \alpha^5 - 624516620 \alpha^4 + 90119120 \alpha^3 - 8601960 \alpha^2 + 492520 \alpha - 12859)$ ],

for  $1/3 \leq \alpha \leq 5/12$ ;

$= [1/562477154400 (1680261559894267527834 \alpha^{16} - 21114326815899849024192 \alpha^{15} + 77668993485702225668880 \alpha^{14} - 139127729454890213869440 \alpha^{13} + 133569608247744951555000 \alpha^{12} - 52089549434211721513536 \alpha^{11} - 31224363559116480921168 \alpha^{10} + 58433658916566896202240 \alpha^9 - 41495374979148593002500 \alpha^8 + 17985529418824426979520 \alpha^7 - 5224402082494305638928 \alpha^6 + 1086007601060137461888 \alpha^5 - 181855389899762397000 \alpha^4 + 26393912063603542080 \alpha^3 - 2788824502549174320 \alpha^2 + 192032353044576576 \alpha - 6397403808895225) / (34783088231 \alpha^{16} - 214666362320 \alpha^{15} + 598699243320 \alpha^{14} - 987322942480 \alpha^{13} + 1051253088340 \alpha^{12} - 728786267664 \alpha^{11} + 296034530792 \alpha^{10} - 24888852560 \alpha^9 - 50147298630 \alpha^8 + 35499452560 \alpha^7 - 13223826616 \alpha^6 + 3314425296 \alpha^5 - 624516620 \alpha^4 + 90119120 \alpha^3 - 8601960 \alpha^2 + 492520 \alpha - 12859)$ ],

for  $5/12 \leq \alpha \leq 4/9$ ;

$= [1/562477154400 (13302404184373014279834 \alpha^{16} - 97562198301360049881792 \alpha^{15} + 311833644882607345412880 \alpha^{14} - 581948900288253272557440 \alpha^{13} + 711300454783028655363000 \alpha^{12} - 602309403277339058473536$

$\alpha^{11}+363303650159511120691632 \alpha^{10}-157902048461612741045760$   
 $\alpha^9+49593343916927043733500 \alpha^8-11252824794730715676480$   
 $\alpha^7+1772810891689831065072 \alpha^6-101398236983231190912 \alpha^5-56203978466601693000$   
 $\alpha^4+22098137313751894080 \alpha^3-3607067312044726320 \alpha^2+313253510006880576 \alpha-$   
 $11785010784997625)/(34783088231 \alpha^{16}-214666362320 \alpha^{15}+598699243320 \alpha^{14}-$   
 $987322942480 \alpha^{13}+1051253088340 \alpha^{12}-728786267664 \alpha^{11}+296034530792 \alpha^{10}-$   
 $24888852560 \alpha^9-50147298630 \alpha^8+35499452560 \alpha^7-13223826616 \alpha^6+3314425296 \alpha^5-$   
 $624516620 \alpha^4+90119120 \alpha^3-8601960 \alpha^2+492520 \alpha-12859)],$

for  $4/9 \leq \alpha \leq 1/2$ ;

$= [1/281238577200 (99330611682389828787 \alpha^{16}-1133745165047646650016$   
 $\alpha^{15}+4952943235515022748280 \alpha^{14}-10263535123723379588160$   
 $\alpha^{13}+5549957248672067065380 \alpha^{12}+25532651361373476701472 \alpha^{11}-$   
 $81154570265561404219608 \alpha^{10}+130588564338716307183360 \alpha^9-$   
 $140563138766606032547070 \alpha^8+108981889267390736556960 \alpha^7-$   
 $62480455475574059691768 \alpha^6+26625378884440231127232 \alpha^5-8351987419402155039420$   
 $\alpha^4+1878157391539838703840 \alpha^3-286981300719821914920 \alpha^2+26676967477902344064$   
 $\alpha-1138730149127308451)/(329134489 \alpha^{16}-3583853104 \alpha^{15}+17652091080 \alpha^{14}-$   
 $51743456240 \alpha^{13}+99651313580 \alpha^{12}-130547634672 \alpha^{11}+114524482072 \alpha^{10}-$   
 $59442800560 \alpha^9+5075529030 \alpha^8+19868317040 \alpha^7-18928357448 \alpha^6+9889374768 \alpha^5-$   
 $3466028020 \alpha^4+850102960 \alpha^3-141924120 \alpha^2+14374256 \alpha-665111)],$

for  $1/2 \leq \alpha \leq 5/9$ ;

$= [1/281238577200 (270757215393451343379 \alpha^{16}-2037689591395993619616$   
 $\alpha^{15}+6136679984304524732280 \alpha^{14}-6636597610597296068160 \alpha^{13}-$   
 $13088471638225862134620 \alpha^{12}+66503612157797753501472 \alpha^{11}-$   
 $139302488214342064219608 \alpha^{10}+189768051326737707183360 \alpha^9-$   
 $185694284233388407547070 \alpha^8+135290586337045736556960 \alpha^7-$   
 $74275614423646559691768 \alpha^6+30673608558565231127232 \alpha^5-9398785742995905039420$   
 $\alpha^4+2075850800914838703840 \alpha^3-312748792907321914920 \alpha^2+28751045602902344064$   
 $\alpha-1216488938189808451)/(329134489 \alpha^{16}-3583853104 \alpha^{15}+17652091080 \alpha^{14}-$   
 $51743456240 \alpha^{13}+99651313580 \alpha^{12}-130547634672 \alpha^{11}+114524482072 \alpha^{10}-$   
 $59442800560 \alpha^9+5075529030 \alpha^8+19868317040 \alpha^7-18928357448 \alpha^6+9889374768 \alpha^5-$   
 $3466028020 \alpha^4+850102960 \alpha^3-141924120 \alpha^2+14374256 \alpha-665111)],$

for  $5/9 \leq \alpha \leq 7/12$ ;

$$= [1/5739562800 (2876907462227260803 \alpha^{16} - 21618001904560426656 \alpha^{15} + 58680033824409843960 \alpha^{14} - 10874998132679338560 \alpha^{13} - 395923995838268237340 \alpha^{12} + 1387272784924406513952 \alpha^{11} - 2714334134341041268056 \alpha^{10} + 3631741520763867413760 \alpha^9 - 3543581003208563592990 \alpha^8 + 2587852112360565267360 \alpha^7 - 1426504309668936473976 \alpha^6 + 591649426375970219712 \alpha^5 - 182025633014900978940 \alpha^4 + 40349471281058953440 \alpha^3 - 6098485624594102440 \alpha^2 + 562144673844748224 \alpha - 23837455620802657) / (329134489 \alpha^{16} - 3583853104 \alpha^{15} + 17652091080 \alpha^{14} - 51743456240 \alpha^{13} + 99651313580 \alpha^{12} - 130547634672 \alpha^{11} + 114524482072 \alpha^{10} - 59442800560 \alpha^9 + 5075529030 \alpha^8 + 19868317040 \alpha^7 - 18928357448 \alpha^6 + 9889374768 \alpha^5 - 3466028020 \alpha^4 + 850102960 \alpha^3 - 141924120 \alpha^2 + 14374256 \alpha - 665111)],$$

for  $7/12 \leq \alpha \leq 2/3$ ;

$$= [1/5739562800 (2931914638206134307 \alpha^{16} - 32058551312728666272 \alpha^{15} + 159775127561476821240 \alpha^{14} - 478960531376017158720 \alpha^{13} + 958860934378384941540 \alpha^{12} - 1343576085662425226976 \alpha^{11} + 1338191372401985837736 \alpha^{10} - 934844636266019930880 \alpha^9 + 429622907213741502690 \alpha^8 - 99545061022464451680 \alpha^7 - 15370015462468410744 \alpha^6 + 21899070288112215744 \alpha^5 - 8574567860937028860 \alpha^4 + 1869733726422317280 \alpha^3 - 236224859150088360 \alpha^2 + 14920397005053888 \alpha - 306783054760033) / (329134489 \alpha^{16} - 3583853104 \alpha^{15} + 17652091080 \alpha^{14} - 51743456240 \alpha^{13} + 99651313580 \alpha^{12} - 130547634672 \alpha^{11} + 114524482072 \alpha^{10} - 59442800560 \alpha^9 + 5075529030 \alpha^8 + 19868317040 \alpha^7 - 18928357448 \alpha^6 + 9889374768 \alpha^5 - 3466028020 \alpha^4 + 850102960 \alpha^3 - 141924120 \alpha^2 + 14374256 \alpha - 665111)],$$

for  $2/3 \leq \alpha \leq 7/9$ ;

$$= [1/318864600 (150015950811091899 \alpha^{16} - 1634619156773175504 \alpha^{15} + 8102397830911420680 \alpha^{14} - 24090211803441173040 \alpha^{13} + 47637944081849901780 \alpha^{12} - 65501143743428042832 \alpha^{11} + 63263556324314898552 \alpha^{10} - 41796876960674138160 \alpha^9 + 16897420676066403330 \alpha^8 - 2025463728086987760 \alpha^7 - 2046501237856283208 \alpha^6 + 1418997571875073008 \alpha^5 - 443571411462199020 \alpha^4 + 72482056540482960 \alpha^3 - 3967591802654520 \alpha^2 - 527535200972784 \alpha + 68676357553969) / (329134489 \alpha^{16} - 3583853104 \alpha^{15} + 17652091080 \alpha^{14} - 51743456240 \alpha^{13} + 99651313580 \alpha^{12} - 130547634672 \alpha^{11} + 114524482072 \alpha^{10} - 59442800560 \alpha^9 + 5075529030 \alpha^8 + 19868317040 \alpha^7 - 18928357448 \alpha^6 + 9889374768 \alpha^5 - 3466028020 \alpha^4 + 850102960 \alpha^3 - 141924120 \alpha^2 + 14374256 \alpha - 665111)],$$

$$\alpha^{11}+114524482072 \alpha^{10}-59442800560 \alpha^9+5075529030 \alpha^8+19868317040 \alpha^7-18928357448 \alpha^6+9889374768 \alpha^5-3466028020 \alpha^4+850102960 \alpha^3-141924120 \alpha^2+14374256 \alpha-665111)],$$

for  $7/9 \leq \alpha \leq 5/6$ ;

$$= [1/200 (46406555513 \alpha^5+30076745795 \alpha^4-19006794310 \alpha^3+1617937030 \alpha^2+844940125 \alpha-59345753)/(329134489 \alpha^5+36626275 \alpha^4-47416790 \alpha^3+27704630 \alpha^2-7058035 \alpha+665111)],$$

for  $5/6 \leq \alpha \leq 1$ .

## 2) Condorcet Efficiency of BR (with $\alpha = k/n$ )

$$CE_{CW}^{\infty}(BR, \alpha, FIAC) = [-1/134400 (410540268461 \alpha^{11}+68290337856 \alpha^{10}-1890950607040 \alpha^9+5562281123840 \alpha^8-10895307579520 \alpha^7+15081068317952 \alpha^6-14952655221504 \alpha^5+10620427234560 \alpha^4-5292864994560 \alpha^3+1761007156480 \alpha^2-351873313792 \alpha+31988483072)/(4032969 \alpha^{11}-10344060 \alpha^{10}+20040630 \alpha^9-49394240 \alpha^8+99197280 \alpha^7-139767264 \alpha^6+140492352 \alpha^5-101312640 \alpha^4+51222600 \alpha^3-17222920 \alpha^2+3459456 \alpha-314496)],$$

for  $0 \leq \alpha \leq 1/8$ ;

$$= [1/116121600 (1052668091602976 \alpha^{16}-2873752619014144 \alpha^{15}+4272609231144960 \alpha^{14}-6345127169884160 \alpha^{13}+10038892987002880 \alpha^{12}-13217647198199808 \alpha^{11}+12962086734012416 \alpha^{10}-9183726384701440 \alpha^9+4574114969149440 \alpha^8-1521630140293120 \alpha^7+304029039362048 \alpha^6-27638765027328 \alpha^5+37273600 \alpha^4-1433600 \alpha^3+38400 \alpha^2-640 \alpha+5)/(\alpha^5 (4032969 \alpha^{11}-10344060 \alpha^{10}+20040630 \alpha^9-49394240 \alpha^8+99197280 \alpha^7-139767264 \alpha^6+140492352 \alpha^5-101312640 \alpha^4+51222600 \alpha^3-17222920 \alpha^2+3459456 \alpha-314496))],$$

for  $1/8 \leq \alpha \leq 1/7$ ;

$$= [1/812851200 (9761447642232104 \alpha^{16}-25585459192553344 \alpha^{15}+35768111967430080 \alpha^{14}-48322455088799360 \alpha^{13}+72086013183839200 \alpha^{12}-93145391738765184 \alpha^{11}+90897475587254336 \alpha^{10}-64319323151923840 \alpha^9+32024146679244720 \alpha^8-10652089317950080 \alpha^7+2128271109124160 \alpha^6-193476640925568 \alpha^5+575542240 \alpha^4-23864960 \alpha^3+692160 \alpha^2-12544 \alpha+107)/(\alpha^5 (4032969 \alpha^{11}-10344060 \alpha^{10}+20040630 \alpha^9-49394240 \alpha^8+99197280 \alpha^7-139767264 \alpha^6+140492352 \alpha^5-101312640 \alpha^4+51222600 \alpha^3-17222920 \alpha^2+3459456 \alpha-314496))],$$

for  $1/7 \leq \alpha \leq 1/6$ ;

$$= [1/4064256000 (6507516258765256 \alpha^{16}-383036306742656 \alpha^{15}+978983871741120 \alpha^{14}-88940076385605760 \alpha^{13}+269967606813588320 \alpha^{12}-426435967871991168 \alpha^{11}+441532739658058048 \alpha^{10}-318286211943539840 \alpha^9+159457664031185520 \alpha^8-53155937254259840 \alpha^7+10628430492167488 \alpha^6-966141627255168 \alpha^5-87966030880 \alpha^4+4776410240 \alpha^3-179854080 \alpha^2+4199104 \alpha-45819)/(\alpha^5 (4032969 \alpha^{11}-10344060 \alpha^{10}+20040630 \alpha^9-49394240 \alpha^8+99197280 \alpha^7-139767264 \alpha^6+140492352 \alpha^5-101312640 \alpha^4+51222600 \alpha^3-17222920 \alpha^2+3459456 \alpha-314496))],$$

for  $1/6 \leq \alpha \leq 1/5$ ;

$$= [-1/4064256000 (36399740821312869 \alpha^{16}-135484639474507344 \alpha^{15}+200669209487633880 \alpha^{14}-97255138458144240 \alpha^{13}-150247064821400820 \alpha^{12}+369597154590741168 \alpha^{11}-420921758642433048 \alpha^{10}+312463050849789840 \alpha^9-158162233175716770 \alpha^8+52928266060509840 \alpha^7-10596924392542488 \alpha^6+962744729205168 \alpha^5+367696618380 \alpha^4-21784800240 \alpha^3+899965080 \alpha^2-23166864 \alpha+279976)/(\alpha^5 (4032969 \alpha^{11}-10344060 \alpha^{10}+20040630 \alpha^9-49394240 \alpha^8+99197280 \alpha^7-139767264 \alpha^6+140492352 \alpha^5-101312640 \alpha^4+51222600 \alpha^3-17222920 \alpha^2+3459456 \alpha-314496))],$$

for  $1/5 \leq \alpha \leq 1/4$ ;

$$= [1/7056000 (132552246694898331 \alpha^{16}-748293358003790256 \alpha^{15}+1926264899049198120 \alpha^{14}-3025551023736287760 \alpha^{13}+3284395514224280820 \alpha^{12}-2653654125536891568 \alpha^{11}+1673579624801332248 \alpha^{10}-840779510496765840 \alpha^9+331607511838372770 \alpha^8-97460228735709840 \alpha^7+19519383563768088 \alpha^6-2344543540033968 \alpha^5+161923351573620 \alpha^4-13962868511760 \alpha^3+833436754920 \alpha^2-30785445936 \alpha+530419424)/(49567630400 \alpha^{16}-288352278272 \alpha^{15}+752425261440 \alpha^{14}-1181716910080 \alpha^{13}+1263251691520 \alpha^{12}-986236532736 \alpha^{11}+588941889536 \alpha^{10}-276340613120$$

$\alpha^9+102040773120 \alpha^8-28745699840 \alpha^7+5797535744 \alpha^6-774638592 \alpha^5+70120960 \alpha^4-6072640 \alpha^3+363840 \alpha^2-13484 \alpha+233$ ),

for  $1/4 \leq \alpha \leq 2/7$ ;

$= [1/21168000 (52034262160844593 \alpha^{16}-664891603502340368 \alpha^{15}+2393105117485386360 \alpha^{14}-4562400298325919280 \alpha^{13}+5661380396424394460 \alpha^{12}-5086581019183167504 \alpha^{11}+3515110544322921544 \alpha^{10}-1907796355947001520 \alpha^9+797291121947630310 \alpha^8-242213977999513520 \alpha^7+48524809049781064 \alpha^6-5469992961682704 \alpha^5+299622714432860 \alpha^4-25524004191280 \alpha^3+1498395896760 \alpha^2-54188171408 \alpha+909683872)/(49567630400 \alpha^{16}-288352278272 \alpha^{15}+752425261440 \alpha^{14}-1181716910080 \alpha^{13}+1263251691520 \alpha^{12}-986236532736 \alpha^{11}+588941889536 \alpha^{10}-276340613120 \alpha^9+102040773120 \alpha^8-28745699840 \alpha^7+5797535744 \alpha^6-774638592 \alpha^5+70120960 \alpha^4-6072640 \alpha^3+363840 \alpha^2-13484 \alpha+233)$ ],

for  $2/7 \leq \alpha \leq 1/3$ ;

$= [1/10584000 (803767637028916007 \alpha^{16}-5030625745653957232 \alpha^{15}+14600217627303773640 \alpha^{14}-26018936358587696720 \alpha^{13}+31787256192061273540 \alpha^{12}-28168168887578634096 \alpha^{11}+18711812014591752056 \alpha^{10}-9525401056249558480 \alpha^9+3784581956228763690 \alpha^8-1194876637232758480 \alpha^7+303389398214038136 \alpha^6-61148121003920496 \alpha^5+9261649149347140 \alpha^4-980545376432720 \alpha^3+71706113775240 \alpha^2-3236711345392 \alpha+67938783928)/(34783088231 \alpha^{16}-214666362320 \alpha^{15}+598699243320 \alpha^{14}-987322942480 \alpha^{13}+1051253088340 \alpha^{12}-728786267664 \alpha^{11}+296034530792 \alpha^{10}-24888852560 \alpha^9-50147298630 \alpha^8+35499452560 \alpha^7-13223826616 \alpha^6+3314425296 \alpha^5-624516620 \alpha^4+90119120 \alpha^3-8601960 \alpha^2+492520 \alpha-12859)$ ],

for  $1/3 \leq \alpha \leq 3/8$ ;

$= [1/10584000 (537773784037346087 \alpha^{16}-3434662627704537712 \alpha^{15}+10111571358071031240 \alpha^{14}-18163805387430397520 \alpha^{13}+22213815320963315140 \alpha^{12}-19552072103590471536 \alpha^{11}+12788245475599890296 \alpha^{10}-6352061838932489680 \alpha^9+2445829473923125290 \alpha^8-748625809797545680 \alpha^7+186248556012294776 \alpha^6-37187494189927536 \alpha^5+5517801209660740 \alpha^4-548562921853520 \alpha^3+36993237960840 \alpha^2-1501067554672 \alpha+27259632583)/(34783088231 \alpha^{16}-214666362320 \alpha^{15}+598699243320 \alpha^{14}-987322942480 \alpha^{13}+1051253088340 \alpha^{12}-728786267664 \alpha^{11}+296034530792 \alpha^{10}-24888852560 \alpha^9-50147298630 \alpha^8+35499452560 \alpha^7-13223826616 \alpha^6+3314425296 \alpha^5-624516620 \alpha^4+90119120 \alpha^3-8601960 \alpha^2+492520 \alpha-12859)$ ],



for  $3/8 \leq \alpha \leq 2/5$ ;

$$= [-1/10584000 (493051411275153913 \alpha^{16} - 3658743622295462288 \alpha^{15} + 12657022391928968760 \alpha^{14} - 27115869612569602480 \alpha^{13} + 40261552179036684860 \alpha^{12} - 43891599096409528464 \alpha^{11} + 36279813564400109704 \alpha^{10} - 23139800561067510320 \alpha^9 + 11479345790076874710 \alpha^8 - 4435023060602454320 \alpha^7 + 1330266251795705224 \alpha^6 - 307890600043672464 \alpha^5 + 54362237190339260 \alpha^4 - 7112613828226480 \alpha^3 + 644649843767160 \alpha^2 - 36185005543568 \alpha + 948186580025) / (34783088231 \alpha^{16} - 214666362320 \alpha^{15} + 598699243320 \alpha^{14} - 987322942480 \alpha^{13} + 1051253088340 \alpha^{12} - 728786267664 \alpha^{11} + 296034530792 \alpha^{10} - 24888852560 \alpha^9 - 50147298630 \alpha^8 + 35499452560 \alpha^7 - 13223826616 \alpha^6 + 3314425296 \alpha^5 - 624516620 \alpha^4 + 90119120 \alpha^3 - 8601960 \alpha^2 + 492520 \alpha - 12859)],$$

for  $2/5 \leq \alpha \leq 3/7$ ;

$$= [-1/756000 (84889320246003917 \alpha^{16} - 601942457348729392 \alpha^{15} + 1998870401088822840 \alpha^{14} - 4126442575657050320 \alpha^{13} + 5925617837835617740 \alpha^{12} - 6272043837139110576 \alpha^{11} + 5056145667145508936 \alpha^{10} - 3161861517144924880 \alpha^9 + 1547515819612581390 \alpha^8 - 593954046764100880 \alpha^7 + 178169023573156616 \alpha^6 - 41429849361294576 \alpha^5 + 7354028307147340 \alpha^4 - 965759629266320 \alpha^3 + 88081540468440 \alpha^2 - 4986650284912 \alpha + 132067086925) / (34783088231 \alpha^{16} - 214666362320 \alpha^{15} + 598699243320 \alpha^{14} - 987322942480 \alpha^{13} + 1051253088340 \alpha^{12} - 728786267664 \alpha^{11} + 296034530792 \alpha^{10} - 24888852560 \alpha^9 - 50147298630 \alpha^8 + 35499452560 \alpha^7 - 13223826616 \alpha^6 + 3314425296 \alpha^5 - 624516620 \alpha^4 + 90119120 \alpha^3 - 8601960 \alpha^2 + 492520 \alpha - 12859)],$$

for  $3/7 \leq \alpha \leq 1/2$ ;

$$= [1/756000 (298789835425997 \alpha^{16} - 6548505782331952 \alpha^{15} + 47139189049859640 \alpha^{14} - 183350486562906320 \alpha^{13} + 458289737298478540 \alpha^{12} - 800685395430853296 \alpha^{11} + 1024618639999330376 \alpha^{10} - 987014931483689680 \alpha^9 + 726889628038821390 \alpha^8 - 412104674322449680 \alpha^7 + 179754194068224776 \alpha^6 - 59781045226362096 \alpha^5 + 14852526786910540 \alpha^4 - 2655616093842320 \alpha^3 + 320405434367640 \alpha^2 - 23210932021072 \alpha + 755096838145) / (329134489 \alpha^{16} - 3583853104 \alpha^{15} + 17652091080 \alpha^{14} - 51743456240 \alpha^{13} + 99651313580 \alpha^{12} - 130547634672 \alpha^{11} + 114524482072 \alpha^{10} - 59442800560 \alpha^9 + 5075529030 \alpha^8 + 19868317040 \alpha^7 - 18928357448 \alpha^6 + 9889374768 \alpha^5 - 3466028020 \alpha^4 + 850102960 \alpha^3 - 141924120 \alpha^2 + 14374256 \alpha - 665111)],$$

for  $1/2 \leq \alpha \leq 3/5$ ;

$$= [-1/756000 (1480995320824003 \alpha^{16} - 10537431717668048 \alpha^{15} + 29747529700140360 \alpha^{14} - 31932325937093680 \alpha^{13} - 38488252923478540 \alpha^{12} + 196171257930853296 \alpha^{11} - 359653088749330376 \alpha^{10} + 417044458983689680 \alpha^9 - 342159559101321390 \alpha^8 + 206915304222449680 \alpha^7 - 93574658626224776 \alpha^6 + 31576833627162096 \alpha^5 - 7801473887110540 \alpha^4 + 1353883250802320 \alpha^3 - 153039783119640 \alpha^2 + 9821679921232 \alpha - 252999884401) / (329134489 \alpha^{16} - 3583853104 \alpha^{15} + 17652091080 \alpha^{14} - 51743456240 \alpha^{13} + 99651313580 \alpha^{12} - 130547634672 \alpha^{11} + 114524482072 \alpha^{10} - 59442800560 \alpha^9 + 5075529030 \alpha^8 + 19868317040 \alpha^7 - 18928357448 \alpha^6 + 9889374768 \alpha^5 - 3466028020 \alpha^4 + 850102960 \alpha^3 - 141924120 \alpha^2 + 14374256 \alpha - 665111)],$$

for  $3/5 \leq \alpha \leq 2/3$ ;

$$= [1/756000 (317927149765997 \alpha^{16} - 3424858332123952 \alpha^{15} + 16729154029139640 \alpha^{14} - 48764396480826320 \alpha^{13} + 93736008977878540 \alpha^{12} - 123374214659173296 \alpha^{11} + 110525429552914376 \alpha^{10} - 62358335832329680 \alpha^9 + 14407791718281390 \alpha^8 + 8290534625550320 \alpha^7 - 9365467622735224 \alpha^6 + 4121250065381904 \alpha^5 - 897334514809460 \alpha^4 + 29914632077680 \alpha^3 + 34131264719640 \alpha^2 - 8170172721232 \alpha + 635599052401) / (329134489 \alpha^{16} - 3583853104 \alpha^{15} + 17652091080 \alpha^{14} - 51743456240 \alpha^{13} + 99651313580 \alpha^{12} - 130547634672 \alpha^{11} + 114524482072 \alpha^{10} - 59442800560 \alpha^9 + 5075529030 \alpha^8 + 19868317040 \alpha^7 - 18928357448 \alpha^6 + 9889374768 \alpha^5 - 3466028020 \alpha^4 + 850102960 \alpha^3 - 141924120 \alpha^2 + 14374256 \alpha - 665111)],$$

for  $2/3 \leq \alpha \leq 3/4$ ;

$$= [1/756000 (135283665503597 \alpha^{16} - 1233136520975152 \alpha^{15} + 4400718841427640 \alpha^{14} - 5614873323834320 \alpha^{13} - 11440953717289460 \alpha^{12} + 65944318192129104 \alpha^{11} - 149787553117626424 \alpha^{10} + 216548431314678320 \alpha^9 - 220919793062006610 \alpha^8 + 165175591145742320 \alpha^7 - 91730122295836024 \alpha^6 + 37815881522559504 \alpha^5 - 11426906845177460 \alpha^4 + 2459815939085680 \alpha^3 - 356388588192360 \alpha^2 + 30881812569968 \alpha - 1194962758124) / (329134489 \alpha^{16} - 3583853104 \alpha^{15} + 17652091080 \alpha^{14} - 51743456240 \alpha^{13} + 99651313580 \alpha^{12} - 130547634672 \alpha^{11} + 114524482072 \alpha^{10} - 59442800560 \alpha^9 + 5075529030 \alpha^8 + 19868317040 \alpha^7 - 18928357448 \alpha^6 + 9889374768 \alpha^5 - 3466028020 \alpha^4 + 850102960 \alpha^3 - 141924120 \alpha^2 + 14374256 \alpha - 665111)],$$

for  $3/4 \leq \alpha \leq 4/5$ ;

$$= [1/756000 (101103978003597 \alpha^{16} - 795636520975152 \alpha^{15} + 1775718841427640 \alpha^{14} + 4185126676165680 \alpha^{13} - 36920953717289460 \alpha^{12} + 114865918192129104 \alpha^{11} - 221539233117626424 \alpha^{10} + 298550351314678320 \alpha^9 - 294721521062006610 \alpha^8 + 217656819945742320 \alpha^7 - 121119610423836024 \alpha^6 + 50640385432959504 \alpha^5 - 15701741481977460 \alpha^4 + 3512082926605680 \alpha^3 - 536777214624360 \alpha^2 + 50123266056048 \alpha - 2157035432428) / (329134489 \alpha^{16} - 3583853104 \alpha^{15} + 17652091080 \alpha^{14} - 51743456240 \alpha^{13} + 99651313580 \alpha^{12} - 130547634672 \alpha^{11} + 114524482072 \alpha^{10} - 59442800560 \alpha^9 + 5075529030 \alpha^8 + 19868317040 \alpha^7 - 18928357448 \alpha^6 + 9889374768 \alpha^5 - 3466028020 \alpha^4 + 850102960 \alpha^3 - 141924120 \alpha^2 + 14374256 \alpha - 665111)],$$

for  $4/5 \leq \alpha \leq 5/6$ ;

$$= [1/28000 (6356730581111 \alpha^5 + 5627499720845 \alpha^4 - 4272086934490 \alpha^3 + 833332667290 \alpha^2 + 180912639955 \alpha - 61394882711) / (329134489 \alpha^5 + 36626275 \alpha^4 - 47416790 \alpha^3 + 27704630 \alpha^2 - 7058035 \alpha + 665111)],$$

for  $5/6 \leq \alpha \leq 1$ .